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An Economic Reformulation of Public Pension Funding Policy

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Current public pension funding policy has arguably failed on both theoretical and empirical grounds. The traditional actuarial approach elides the risk-return tradeoff at the heart of finance economics and has resulted in steadily rising contribution rates, instead of a sustainable steady state. We propose an economic reformulation of funding policy based on steady-state analysis of the fundamental equations of motion for pension asset and liability growth, incorporating both an expected return on risky assets and a low-risk discount rate for liabilities. Our steady-state result simultaneously conveys the benefit of risky investment and the cost of the associated risk. We integrate our analysis into a simple social welfare function to re-examine the basis for pre-funding and elucidate the net benefits of using risky assets to defray contributions. We also formally derive a family of transition policies for convergence to the expected steady state. We illustrate how the parameters of our proposed policy can be adjusted to manage the tradeoff between long-run contribution rate risk and short-term responsiveness. We believe our analysis provides the basis for reformulating contribution policy in a way that better supports sustainability and coherently conveys the tradeoffs consistent with finance economics.

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AN ECONOMIC REFORMULATION OF PUBLIC PENSION FUNDING POLICY

October 26, 2022

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ABSTRACT: Current public pension funding policy has arguably failed on both theoretical and empirical grounds. The traditional actuarial approach elides the risk-return tradeoff at the heart of finance economics and has resulted in steadily rising contribution rates, instead of a sustainable steady state. We propose an economic reformulation of funding policy based on steady-state analysis of the fundamental equations of motion for pension asset and liability growth, incorporating both an expected return on risky assets and a low-risk discount rate for liabilities. Our steady-state result simultaneously conveys the benefit of risky investment and the cost of the associated risk. We integrate our analysis into a simple social welfare function to re-examine the basis for pre-funding and elucidate the net benefits of using risky assets to defray contributions. We also formally derive a family of transition policies for convergence to the expected steady state. We illustrate how the parameters of our proposed policy can be adjusted to manage the tradeoff between long-run contribution rate risk and short-term responsiveness. We believe our analysis provides the basis for reformulating contribution policy in a way that better supports sustainability and coherently conveys the tradeoffs consistent with finance economics.

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I. Introduction

Most state and local employees are enrolled in final-average-salary defined benefit pension plans. These plans rely on taxpayer and member contributions and investment returns to pay for benefits. Under existing actuarial funding policy, the goal is to fully fund benefits over the course of workers' careers. To achieve this goal, plans must make a host of predictions about the future to set adequate contribution rates. If reality falls short of their predictions, the government sponsor must make up the difference with additional contributions. Unfortunately, existing actuarial funding policy has resulted in contributions that have fallen short of the amount needed to cover promised benefit payments resulting in deteriorating funding ratios, steeply rising government contributions, and reduced benefits for new public workers.

Given the scale of public pension promises, pension sustainability not only has big implications for millions of public workers' retirement security but also for government budgets and future generations of workers and taxpayers. Yet, the concept of sustainability has not been clearly defined, nor have the related policy tradeoffs been comprehensively described. We contend that current actuarial funding policy is part of the problem as it does not adequately convey the risk-return tradeoff and has, as a result, arguably failed to deliver either sustainability or intergenerational equity, however they are defined.

At the heart of the issue with actuarial funding is a puzzle it seemingly cannot solve. It is generally agreed among economists that pension liabilities should be discounted at a low-risk rate corresponding to the guaranteed nature of the benefits promised, at least for reporting purposes. But it remains an open question how proper discounting of liabilities should inform funding policy. Actuarial funding policy sets contributions equal to the normal costs plus amortization of any pension debt. However, normal costs must logically be discounted by the same rate as liabilities, since they are mathematically linked. Consistently discounting normal cost by the low-risk rate would dramatically raise contributions, compared to standard actuarial practice of discounting by the expected (or assumed) return on risky assets. Unless the pension plan decides to invest only in low-yield, risk-free assets – in which case contributions would indeed have to be dramatically elevated – there seems to be no way to fit the square peg of proper liability discounting into the round hole of actuarial funding policy.

The core problem is that actuarial funding formulas are formally deterministic[1] and ill-suited to conveying both the benefits and costs of investment in risky assets. This requires both the expected return and the risk-free return, as the risk premium is the gap between the two. Moreover, the actuarial approach is not embedded in an optimizing framework that would clearly pose the policy tradeoffs between risk and return bound up in the asset accumulation target. In this paper, we propose an economic reformulation of public pension funding policy that helps resolve this dilemma and coherently conveys the tradeoffs consistent with finance economics.

Our economic reformulation derives from the steady-state analysis of the fundamental equations of motion for assets and liabilities. The key policy decision is to set the target funded ratio, but to do so honestly, as a proportion of properly discounted liabilities. In so doing, we fold into the target contribution rate both the expected return on risky assets and the low-risk discount rate for liabilities. Our result simultaneously conveys the benefit of risky investment and the cost of the associated risk, as both are reflected in the risk premium. We integrate this result into a

[1] GASB requires plans to provide some sensitivity analysis for liabilities, and a few plans do so for contribution rates too.

simple social welfare function to clearly convey the risk-return tradeoff that should help guide the choice of a target funded ratio.

One might expect the pursuit of risky returns to enhance the net benefits from pre-funding, and that turns out to be true at the policymakers' optimal asset allocation for an intriguing reason: the convexity of long-run contribution risk with respect to the expected return. This finding supplements the traditional basis for pre-funding, the gap between the pay-go rate and normal cost rate and may even outweigh it. This could be particularly important since properly discounting liabilities narrows the gap between pay-go and normal cost, shrinking the traditional basis for pre-funding. We should emphasize, however, that the pre-funding rationale based on exploiting risky investments may reflect excessive risk-tolerance on the part of policymakers. If so, we would have the ironic result that the case for pre-funding – considered the course of prudence – may rest largely on imprudent attitudes toward risk.

We also formally derive a family of transition policies for convergence to the expected steady state. We illustrate how the parameters of our proposed policy might be adjusted to manage the tradeoff between long-run contribution rate risk and the speed of adjustment toward asset and contribution targets.

Although the analysis of steady states oversimplifies actual systems – even in expected value – since the parameters themselves never settle into steady states, the approach offers insights akin to other simple economic models. Steady state analysis lays out the characteristics of the system's trajectory, even if it is aiming at a moving target. In sum, we believe the economic approach offers a basis for reformulating contribution policy in a way that addresses shortcomings in the actuarial approach.

Plan of the Paper

We begin with a formal definition of sustainability as a funding policy that generates a steady state in the contribution rate and asset-to-payroll ratio. We provide a general derivation of such steady states from the fundamental laws of motion for asset and liability growth, for any given funding target and the parameters of the system – including both the expected return on risky assets and the risk-free discount rate for liabilities.

The traditional actuarial target of fully funding reported liabilities (wrongly discounted) is a special case of our steady-state analysis, as are recent proposals to arbitrarily stabilize pension debt (properly discounted) at current levels (e.g., Lenney, Lutz, and Sheiner (2019a; 2019b). In our general formulation, the funding target is a policy choice, which governs the steady-state contribution rate. We calibrate these relationships with data from the Boston College Public Plans Database (PPD) to illustrate how many plans are making sufficient contributions to sustain current asset ratios under varying assumptions about the return on investments.

Building on previous work (Costrell, 2018a, Costrell and McGee, 2020), we find that the steady-state contribution rate is a blend of the pay-go rate and the (properly discounted) normal cost rate, weighted by the target funded ratio, and partially defrayed by the risk premium between the expected return and the risk-free rate. Among other insights, this relationship shows that under a deterministic interpretation, the steady-state contribution rate can fall well below the normal cost rate, due to implicitly assumed arbitrage profits represented by the gap between the expected return and the risk-free rate. More generally, this gap – the risk premium – mirrors the cost of risk and helps clarify in a formal and precise fashion the tradeoff between the cost of risk and the benefit of lower expected contributions that underlies the choice of asset allocation and accumulation target, a theme to which we return in a later section.

Following our analysis of steady states, we examine the conditions for convergence. Steady states are of less interest if the system does not converge. Moreover, we believe the convergence path should be smooth by design, in contrast to conventional amortization schedules, especially closed interval schedules with cliff funding. Formally, we consider contribution policies where the rate adjusts incrementally to gaps between the steady-state and current values of the two state variables: the contribution rate and asset ratio. The conditions for convergence are surprisingly non-trivial, even in this simple class of deterministic models. We find that positive adjustment parameters are required for both gaps to ensure convergence, and we fully characterize the policy-relevant (non-oscillatory) range of those parameters. This generates a novel contribution policy outside of steady state, differing conceptually and materially from the standard actuarial policy of normal cost plus amortization. After isolating the range of appropriate adjustment parameters, we illustrate transition paths to steady state under current funding targets. We then model transition paths under stochastic returns, showing the widening spread of likely contribution rates over time. We show how the adjustment parameters should be modified, compared to the deterministic case, to manage this risk.

We next turn to the question of how to choose the funding target, along with the target asset return. We posit a semi-formal social welfare function to convey the tradeoffs between intergenerational burden sharing, the pursuit of risky returns, and the social cost of risk bearing. We integrate the steady-state contribution rate and the relationship between the risk premium and risk into this optimization framework. By analyzing the joint optimum for the target asset return and funded ratio, we re-examine the basis for pre-funding, incorporating the benefit of risky investment, net of the social cost of risk. Finally, in concluding remarks, we speculate on how our analysis might guide practical policy choices.

II. STEADY STATE ANALYSIS

Sustainability – the idea of something being sustained – raises the question of what that something (or more than one something) is for pension policy. It seems natural to identify the contribution rate as the key variable that one would want to stabilize and to do so at a level that would stave off risk of insolvency, rating agency downgrades for the taxing authority, crowd-out of other necessary public services, or some other form of fiscal distress. Judging by the fact that contribution rates have been generally rising since around 2000, it does not seem that current actuarial funding models have succeeded in securing this notion of sustainability.

We propose a return to first principles by formally defining sustainability as a steady state in the contribution rate and funded ratio. There are, of course, many such steady states, including, at one extreme, the pay-go rate, at zero funding, and, conversely, many degrees of pre-funding, with their corresponding contribution rates. Thus, the nature of the steady state depends on the goals of the policy, as well as the plan parameters and assumptions, most notably the rate of return. In the next subsections, we lay out steady-state analysis that derives from the fundamental laws of motion of a plan's assets and liabilities to explore the relationship between the steady-state contribution rate and these factors.

Steady State Condition for Contribution Rate and Asset Targets

The proximate determinant of the steady state contribution rate is the asset target. As we will show, the asset target determines that portion of benefits to be covered by investment income, leaving the rest for the steady state contribution rate. Liabilities only enter the picture as a benchmark for the asset target, linked by the target funded ratio. Thus, we consider the asset target first, generating insights of its own, and then bring in liabilities in the next subsection.

There are two sources of pension funding and two uses: contributions and investment income go to cover the payment of benefits and the accumulation of assets. Of these four flow variables, the stream of benefit payments is exogenous to our analysis (determined by the tiered benefit formulas and workforce assumptions), and investment income is governed by the sequentially determined stock of assets and the series of annual returns. This leaves the series of contributions and that of asset accumulation as mechanically linked. That is, the *funding policy is simultaneously a contribution policy and an asset accumulation policy*.

Formally, this relationship is captured in the fundamental asset growth equation:

$$(1)\ A_{t+1} = A_t(1+r_t) + \boldsymbol{c_t} W_t - c^p{}_t W_t\,,$$

where A_t denotes assets at the beginning of period t, r_t is the return in period t, W_t is payroll, while $\boldsymbol{c_t}$ and $c^p{}_t$ are the contribution and benefit payment rates, respectively, as proportions of payroll (Table 1 lists notation). Assets grow by investment earnings, plus contributions, net of benefit payments. Equation (1) is simply an accounting identity. To give it economic content, for sustainability analysis, we need to specify a funding policy to drive $\boldsymbol{c_t}$. Given returns and benefit payments, the contribution policy sets asset growth. We will spell out our approach to the choice of contribution policy below, but even before delving into the specifics, equation (1) helps focus on the fundamental tradeoffs among these policies.

It will be useful to re-express equation (1) in terms of the ratio of assets to payroll, $a \equiv (A/W)$. Dividing through (1) by W_t, and denoting the growth rate of payroll by g, we have:

$$(1')\ a_{t+1}(1+g_t) = a_t(1+r_t) + \boldsymbol{c_t} - c^p{}_t\,.$$

The big picture here can be illuminated by examining the steady-state relationship between the contribution rate and assets. In steady state, the growth rate of assets must equal that of payroll, so the asset ratio is constant, $a_{t+1} = a_t = a^*$. Removing the time subscript for the steady-state values of the benefit payment rate c^p, the rate of return r, and the payroll growth rate g, we have the relationship between the steady-state contribution rate and asset ratio:

(1*) $\boldsymbol{c^*} = c^p - (r - g)a^*$.

The interpretation is straightforward: benefit payments are covered by a mix of contributions and investment income (net of growth), where the mix is determined by the funding policy. Under a policy of pay-go, where no assets are accumulated ($a^* = 0$), the contribution rate must cover the benefits payment rate c^p. Under a policy of pre-funding, to one degree or another, the goal is to accumulate a certain asset level, a^*, so the income from those assets (net of growth) can help fund benefits, ultimately reducing reliance on contributions.

One very modest test of sustainability is to consider whether current contribution rates are sufficient to sustain a steady state at *current* asset levels. To be sure, that is not the goal of current policies, which are attempting to raise asset levels to amortize pension debt. But if we consider the minimal target of $a^* = a_0$, would the current contribution rate, c_0, need to rise to sustain the asset level? Is $c_0 < c^*(a_0)$?[2]

Let us consider the trends and magnitudes of the relevant variables. Figure 1 depicts the aggregate values of $\boldsymbol{c_t}$ and c^p_t for FY01 – FY20, of the 119 state and 91 local plans in the PPD, which account for 95 percent of state and local pension assets and members in the U.S. As mentioned above, the contribution rate, as a percent of payroll, has been steadily climbing since the turn of the century, from about 12 percent to 27 percent.[3]

The benefit (or "pay-go") rate has also trended up, due in part to benefit increases enacted in the 1990s and early 2000s, but largely due to the aging workforce and the falling number of

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[2] Equivalently, using (1*): is $a_0 < a^*(c_0)$? If so, then a policy of holding contributions at the current rate of c_0 would lead to a continual draw-down of assets, following the dynamic of *(1')*. That is, the current contribution rate would lead to insolvency. This is a special case of the convergence analysis discussed below.

[3] This includes employer and employee contributions. The FY20 mix is 20 percent and 7 percent, respectively.

actives per retiree. The pay-go rate has risen from 20 percent to 38 percent but may now be leveling off.[4] It is important to note that throughout this period the benefit rate exceeds the contribution rate by a large margin, exceeding 10 percentage points since 2010. That is, the primary cash flow (i.e., excluding investment income) is negative, due to some combination of plan maturity and possibly some contribution shortfall (the question we are considering in some form). Thus, if assets were to be depleted, contributions would have to jump to cover benefits.

Figure 2 depicts the asset ratio $a \equiv (A/W)$ from the same dataset. This has fluctuated with market returns, along with trends in benefit payments and contributions, but in recent years assets have hovered around a multiple of 5 times covered payroll.

With these data for a_0 and c^p, along with typical plan assumptions of g = 3% and r = 7%, we calculate $c^*(a_0) = c^p - (r - g)a_0 = 0.38 - (0.07 - 0.03) \times 5 = 0.18$. This is less than the current contribution rate, $c_0 = 0.27$. Thus, taken at face value, this would suggest that, *in the aggregate*, the current configuration is sustainable, and, indeed, that contribution rates could fall while still supporting current asset ratios. Of course, this depends on a host of assumptions, not least of which are the assumed rate of return and growth rate – specifically, the gap between the two. As long as $(r - g)$ exceeds about 2 percent (e.g., $r > 5$ percent for $g = 3$ percent), the current overall contribution rate could sustain a_0, under this simple analysis.

This picture also generally holds for the individual plans in the PPD database. Using each plan's assumed return (the vast majority lie between 7.0 and 7.5 percent for FY20), we find that in 158 of the 188 plans for which $c^*(a_0)$ can be calculated, the contribution rate exceeds that value.[5] This also holds for 69 of the 79 largest plans, with assets exceeding $10 billion.

[4] Lenney, Lutz, and Sheiner (2019a; 2019b) project that the benefit rate will peak over the next decade and decline thereafter, as recent hires, in less generous tiers, enter retirement and beneficiaries of more generous tiers die off.
[5] The assumed growth rate for payroll is only available in the PPD for 76 plans. Of those, the vast majority lie between 2.75 and 3.5 percent, so we set the growth rate at 3.0 percent for the calculation of $c^*(a_0)$ in all plans.

This result, however, is sensitive to the assumed return, or, more precisely, the assumed gap between r and g. Reducing each plans' assumed return to 5.0 percent (while holding g = 3 percent) changes the picture. Under this assumption, the contribution rate for most plans (107 of the 188 plans, and 48 of the largest 79 plans) is too low to sustain the current asset ratio. As this exercise illustrates, it is important to bear in mind that the steady states we examine are, at best, steady states in the expected value of contributions, with significant risk in the actual outcomes.

Indeed, looking back over the time series depicted, even though c_0 has consistently exceeded $c^*(a_0)$ under the assumed gap between r and g, the asset ratio has not risen. Despite ever-rising contribution rates, the attempt to raise the asset ratio has generally failed. This not only indicates faulty assumptions; more fundamentally, it points to a failure of contribution policies to self-correct – a topic we return to below under the subject of convergence. But first, we turn to the role of liabilities in setting asset targets.

Steady State Condition for Liabilities

We begin with the fundamental growth equation for liabilities:

$$(2)\ L_{t+1} = L_t(1+d) + c^n_t W_t - c^p_t W_t\ ,$$

where L_t denotes accrued liabilities at the beginning of period t, d is the discount rate, and c^n_t is the normal cost rate, the rate at which new liabilities accrue, as a percent of payroll. Liabilities grow by the interest on past liabilities, plus newly accrued liabilities, net of benefit payments that extinguish prior liabilities. Equation (2) is analogous to the asset growth equation (1), but with some key differences:

First, the formulation in (2) allows for a distinction between the discount rate d and the assumed (or expected) rate of return on assets r. Standard actuarial practice, of course, has traditionally discounted by r. By contrast, finance economics has consistently made the case that guaranteed benefits should be discounted by interest rates of correspondingly low-risk bonds, at least for accounting purposes (Novy-Marx & Rauh, 2009, 2011; Brown & Wilcox, 2009; Biggs, 2011). If asset accumulation and projections thereof reflect actual and assumed returns on a higher-risk pension fund portfolio, this raises the question of how a dual rate system should play out in contribution policy. In the previous subsection, focused on asset accumulation, the steady-state contribution policy depended only on r and not on d. We consider below how the consideration of liabilities, discounted at $d < r$, should factor into contribution policy.

The second difference between the liability growth equation (2) and the asset accumulation equation (1) is the role of c^n_t, the normal cost rate, vs. $c_{t,}$, the contribution rate. The normal cost rate is independent of the contribution policy. It is determined by the benefit formula, the cohort's assumed separation probabilities over its members' careers, and (importantly) the discount rate.[6] The normal cost rate may be used to help determine contributions (as in standard actuarial policy), but not the other way around. This means equation (2) is logically prior to the asset accumulation and contribution equation (1). In our model, equation (2) will feed into (1*) by tying the asset target, a^*, to liabilities.

It will be useful to express (2) in the state variable $\lambda = L/W$ = liabilities/payroll, using the same steps as in the derivation of (1′):

$$(2') \ \lambda_{t+1}(1+g_t) = \lambda_t(1+d) + c^n_t - c^p_t\,.$$

If we take the benefit formula and demographic/worklife assumptions as exogenous, along with d and g, then so are c^n and c^p. Thus, we can readily derive the steady-state liability ratio:

$$(2^*) \ \lambda^* = (c^p - c^n)/(d - g).$$

[6] It also depends on the specific actuarial cost method for allocating liabilities between past and future accruals. To fix ideas, we have in mind the standard entry age normal cost method.

This expression has a simple interpretation. First note that the present value of future payroll in steady state is $W_t/(d-g)$, using the formula for a growing perpetuity. The PV of future benefit payments and liability accruals (normal costs) are, respectively, fractions c^p and c^n of the PV of future payroll. Thus, equation (2*)'s steady-state liability ratio, λ^*, represents the difference between the PV of future benefit payments and normal costs, scaled to current payroll.[7]

Figure 3 depicts the aggregate liability ratio, drawing again on the PPD, where the liabilities are reported based on each plan's assumed return, *r*. That ratio (depicted by the red curve) has gradually risen from about 4.6 in FY01 to about 7.2 in FY20, a rise of 56 percent. Several factors have contributed to this trend, including reductions in the assumed return and a rise in the ratio of retireds to actives.[8]

Liabilities are much higher when discounted at a low-risk rate *d*, instead of *r*. Estimates vary regarding the magnitude of the impact. Here, we consider the liability estimates of the Federal Reserve Board of Governors, depicted by the black curve in Figure 3.[9] Comparing these estimates with the reported values of the PPD suggest that properly discounted liabilities are about 60 percent higher.[10] By this measure, the liability ratio has risen from about 7.3 in FY01 to about 12.3 in FY20, a rise of 67 percent.

Tying this together with the previous subsection, on asset accumulation, the actuarial goal of fully funding reported liabilities would raise the asset ratio from about 5 times payroll to 7.

[7] This follows from the basic identity that the PV of all future benefits equals the PV of benefits yet to be accrued (the PV of future normal costs) plus the PV of benefits previously accrued but not yet paid. The latter term is the accrued liability, so it equals the difference between the PV of all future benefits and the PV of future normal costs.
[8] Benefit changes have also affected the trends, but in no simple fashion, as many plans raised benefits in the early 2000's and then cut them for new hires in the 2010's. Comparing the liability ratios with the calculated values of λ^* for FY01, FY10, and FY20, we find these values match for FY01 (4.6 vs. 4.5), but for FY10 and FY20, the liability ratios exceed the calculated values, 5.7 vs. 4.3 and 7.2 vs. 6.1, respectively. There are many potential explanations for these gaps, but they would be consistent with plans that are beyond mature, rather than in steady state.
[9] The denominator in the ratio depicted is the PPD payroll series.
[10] Lenny, Lutz, Schule, and Sheiner's estimates of the funded ratio, using reported liabilities (2021 Table 1) and rediscounted liabilities (2021 Table A7) imply that the latter is about 80 percent higher.

Although this goal has proven challenging, it still falls well short of matching true liabilities. The true funded ratio, upon accumulating assets of 7 times payroll, would be about 7/12, or 58 percent. Considering the rise in contributions that would be needed to achieve this goal (examined in the next section), this may well be the limit of what is politically feasible or socially optimal under a social welfare function of the type we introduce in a later section.

In any case, a more accurate label for the current policy would be something like "60 percent funding" (of true liabilities) rather than "full funding." More generally, as we will formalize below, the way to integrate risk-free discounting of liabilities into a policy of accumulating risky assets with higher expected returns is to set the target funded ratio relative to true liabilities. That target may well be less than 100 percent, but it would have the virtue of being accurate and, as we will show, such a policy will integrate the costs of risky investment with the benefits of reduced expected contributions.

Target Funded Ratio and the Steady State Contribution Rate

The natural link between our steady-state analysis of asset accumulation and liabilities is to tie the asset goal, a^*, to liabilities. We here consider the general goal of a target funded ratio, f^*, including both "full" actuarial funding, and such putative standards as "the 80 percent rule."[11] Setting the asset goal of $a^* = f^*\lambda^*$, and, for the moment, following the actuarial convention of $d = r$, we find, from (1*) and (2*):

$$(3)\ \boldsymbol{c^*} = c^p - (r - g)f^*\lambda^* = c^p - f^*(c^p - c^n) = (1 - f^*)c^p + f^*c^n.$$

As the funded goal varies from zero to full funding, the steady-state contribution rate varies from the pay-go rate to the normal cost rate, with a weighted average of the two for intermediate

[11] See Costrell, 2018a, where equation (3) was previously derived.

funding targets. Thus, full actuarial funding is a special case, where the steady-state contribution rate is c^n (discounted at r), reached upon completion of the amortization schedule.

Let us now consider the steady-state implications of a dual rate system: discount rate d for liabilities and assumed return r on assets. We then have:

$$(3') \ \boldsymbol{c^*} = c^p - (r-g)f^*\lambda^* = c^p - [(r-g)/(d-g)]f^*(c^p - c^n).$$

As before, if the funding goal f^* is zero, the contribution target is pay-go, and as f^* is set higher, the contribution target falls. Our question here is the impact on c^* of reducing d below r. We have already seen from (1*) that the only avenue for a drop in d to affect c^* is through its impact on the asset target a^*. Since we are considering asset goals of the form $a^* = f^*\lambda^*$, this means that a drop in d below r would raise the target contribution rate through a rise in the liability ratio λ^* unless it is offset by a reduction in the target funded ratio f^*.

If, for example, our funding goal is to merely maintain the current asset ratio, $a^* = a_0$, then the rise in λ^* from revaluation at d would, in effect, be completely offset by an implicit drop in the target funded ratio f^*.[12] Under this simple goal, the distinct role of d drops out of *(3')*, and we are back at (1*) with $\boldsymbol{c^*} = c^p - (r-g)a_0$. Setting d to a low-risk rate for the valuation of liabilities is here purely an accounting and reporting measure, unrelated to contribution policy.

More generally, let us consider the implications of setting $d < r$ when f^* is a deliberately chosen target (as discussed in a later section), rather than an artifact of maintaining the status quo. The first implication is that under a full-funding policy, $f^* = 1$ (or anywhere near full), $c^* < c^n$: contributions will not cover normal costs (properly discounted). Formally, *(3')* implies

$$(3'') \ \boldsymbol{c^*} - c^n = (c^p - c^n)(1 - [(r-g)/(d-g)]f^*) < 0, \text{ for } f^* > [(d-g)/(r-g)].$$

[12] Specifically, the rise in λ^* effectively reduces f^* to a_0/λ^*, so (3') simplifies to $\boldsymbol{c^*} = c^p - (r-g)a_0$. This is implicit in the Lenney, Lutz, and Sheiner (2019a; 2019b) model. This explains why the contribution rate in their model is essentially independent of d, despite their claim that setting $d < r$ makes their model conservative.

To fix magnitudes here, consider the values we have been using, $r = 0.07$ and $g = 0.03$, along with $d = 0.04$ (a typical discount rate used in private pension accounting). The critical value of f^* in the expression above is then 25 percent. For any target funded ratio exceeding 25 percent of true liabilities, steady-state contributions need not cover the true normal costs (discounted at d). This contrasts starkly with standard actuarial funding schedules, under which contribution rates drop to (but not below) reported c^n (discounted at r) upon reaching full funding.

The point can be illuminated by re-writing *(3′)* and simplifying to obtain:[13]

$$(3^*)\ \boldsymbol{c^*} = c^p - (d - g)f^*\lambda^* - (r - d)f^*\lambda^* = (1 - f^*)c^p + f^*c^n - (r - d)f^*\lambda^*.$$

Comparing with (3), where $d = r$, we have a higher (rediscounted) normal cost rate, but the third term, which is negative, is new. Under a deterministic interpretation of c^* and r, this represents the implicitly assumed arbitrage profits between the return on accumulated assets and interest on covered liabilities. These assumed arbitrage profits help defray the higher normal costs, in lieu of contributions that might otherwise be required.

Alternatively, if c^* and r are understood to be expected values of risky variables, the last term may be interpreted as the risk premium on the portfolio. While this reduces the expected contribution rate, it simultaneously mirrors the implicit cost of risk borne by the sponsoring government. This expression nicely captures the tradeoff between risk and return, to which we return in a later section.

[13] We can drop the assumption of steady state in λ and obtain an expression with the same pieces and the same interpretation: $\boldsymbol{c}_t = (1 - f^*)c^p + f^*c^n - (r - d)f^*\lambda_t$. This generalization of (3*) loosens the condition that assets and liabilities grow at the rate g; we require only that they grow at the same rate as each other, so that f is constant at f^*.

III. CONTRIBUTION POLICY FOR CONVERGENCE TO STEADY STATE

Although steady-state calculations are instructive, they are not compelling unless there is a dynamic process that converges toward a steady state. Moreover, that dynamic process, once determined, informs us of the expected costs along a transition path to the target asset ratio a^*. Of course, the steady state is always a moving target, as the parameters c^p, r, and g vary over time, but we can analyze whether the system moves in the right direction, taking these parameters as constants, at their projected values.

In assessing the path to a^*, we need not concern ourselves with the dynamics of the liability ratio, λ_t, given in (2'). Once we determine the steady-state liability ratio, λ^* from (2*), and choose the target funded ratio f^*, we have the target asset ratio $a^* = f^*\lambda^*$. This is all we need to map out a transition path for a_t, using equation (1') and a contribution policy c_t to be specified. As we will show, even with these simplifications, the determination of a smoothly convergent contribution policy is non-trivial.

Convergence is not automatically assured, as can be discerned by considering the asset accumulation equation (1′) alone (before adding in a contribution policy equation). To simplify notation, let $R = 1+r$, $G = 1+g$, and re-express (1′) as:

$$(1'')\ a_{t+1} = a_t(R/G) + (\boldsymbol{c_t} - c^p)/G.$$

For $R > G$ (as usually assumed), the coefficient on the prior value of the state variable a exceeds one, which is destabilizing. For example, suppose we consider a policy that sets the contribution rate to some target rate and holds it constant.[14] Unless that target rate corresponds to the steady-state value for maintaining the *current* asset ratio, the system will diverge. Stated alternatively, suppose one aims at an asset ratio $a^* \neq a_0$, and immediately sets $c = c^*$ (using (1*)), jumping up

[14] Many states set fixed rates in statute, instead of actuarially determined rates. Similarly, the Lenney, Lutz, and Sheiner (2019a; 2019b) policy simulation sets c equal to a steady-state value and holds it there.

or down from c_0, and holding it there. The system will then move away from a^*, rather than toward it. If a^* is set greater than a_0, then a_t, will shrink further away from a^*, and conversely if a^* is set lower than a_0.[15]

The reason is straightforward. Setting a higher a^* means setting a lower c^* for $r > g$ (see equation (1*)), since one expects to rely on higher investment income, in lieu of contributions, to cover benefits. But since assets are not yet at that higher level of a^*, the investment income falls short of that which would obtain in the steady state to which one aspires. Thus, by prematurely setting contributions at the correspondingly low level, c^*, one embarks on a path of asset decumulation. And conversely for $a^* < a_0$.

So, what would a contribution policy look like that converges to a steady state targeted at a^* with contributions c^*? It might be thought that an adjustment process that gradually closes the *contribution gap* between c^* and c_t, rather than a sudden jump to c^*, would do the job, but as we shall see below, it will not. The reason, as would be suggested by the discussion above, is that the contribution required to cover benefits depends on the *asset gap* between a^* and a_t. Alternatively, then, one might suppose that an adjustment process for contributions based on the asset gap would do the job. However, as we shall see, that will not suffice either. For a convergent path, we will show that the policy should adjust contributions based on *both* gaps, between c^* and c_t and between a^* and a_t, in combinations to be derived below.

Before doing so, note that the policy we are deriving differs not only from a discrete jump to c^*, but also from the trajectory of actuarial funding policy. The actuarial payment schedule typically sets either a constant percent of payroll, or ramps up to such a rate, and then

[15] Formally, the solution is $a_t = a^* + (R/G)^t(a_0 - a^*)$, which continually magnifies any initial deviation from a^*.

falls off a cliff at the end of the amortization period, once full funding is expected to be achieved.[16] The policy we derive below aims to converge smoothly on a steady state.

Specifically, consider a contribution policy that starts by specifying a target asset ratio, a^* (more on how that might be chosen, in a later section), and calculates the corresponding steady-state contribution rate c^*, using (1*) above. We then annually adjust the contribution rate based on the gaps between the target and actual values for assets and contributions:

$$(4) \quad c_{t+1} = c_t + \beta(c^* - c_t) + \gamma(a^* - a_t), \text{ where } \beta \in (0,1).$$

Along with (1″), we have a simple system of two linear difference equations to be analyzed using standard methods. We derive the bounds on β and γ needed for convergence in the Appendix.

The first convergence condition (see Appendix) is $\gamma > \beta(R - G) \equiv \gamma_{min}$. This is positive for $R > G$, thereby showing formally what was asserted above: a piece of the adjustment mechanism must be based on the asset gap, not just that of the contribution rate. The logic is straightforward. Suppose the contribution rate is already at its target c^*, but the asset level is below the target a^*. Then contributions will have to rise in the short run to accumulate more assets, before eventually dropping back down toward c^*.

The second convergence condition, $\gamma < G - R(1 - \beta) \equiv \gamma_{max}$, implies that the adjustment mechanism must include the contribution gap, too, $\beta > 0$. Formally, since we must have $\gamma_{max} > \gamma_{min}$, this requires $\beta > (R - G)/G > 0$. The logic here is also straightforward. If assets are at their target ratio, but the contribution rate is below c^*, then c_t needs to rise.

As our discussion above suggests, the convergence to steady state may not be monotonic. Indeed, it may not only reverse direction once (asymptotically monotonic), it may be oscillatory. The condition for non-oscillatory behavior, also given in the Appendix, is

[16] This refers to "closed interval" amortization. "Open interval" amortization extends the payoff date each year. Although commonly used in times past, open interval is no longer recommended by GASB.

$\gamma < G[(R/G) - (1 - \beta)]^2/4 \equiv \gamma_{m/o}$, where the subscript *m/o* denotes the boundary between monotonic and oscillatory. It can be shown that for $\gamma_{max} > \gamma_{min}$, $\gamma_{m/0}$ falls between the two.

Thus, the asymptotic behavior of the system varies with the range of γ as given in Table 2. Figure 4 illustrates the combinations of β and γ that correspond to these asymptotic behaviors for r = 7% and g = 3%. In general, it seems reasonable to presume that policymakers would prefer non-oscillatory convergence. Thus, the relevant combinations of β and γ would lie between γ_{min} and $\gamma_{m/o}$, depicted by the black and blue curves in Figure 4.

Deterministic Simulations

Armed with these analytics, we illustrate the dynamic paths for contributions and assets under plausible policies. Taking the representative plan assumptions given above, $R = 1.07$, $G = 1.03$, $c^p = 0.38$, $c_0 = 0.27$ and $a_0 = 5$, we set the target ratio $a^* = 7$. As discussed in the previous section, this increase of 40 percent above a_0 would accumulate approximately the assets needed for "full" actuarial funding (discounted at the expected return), or about 60 percent of true liabilities (discounted at a low-risk bond rate). Equation (1*) gives us $c^* = 0.38 - (0.07 - 0.03) \times 7 = 0.10 < c_0 = 0.27$, thus allowing eventually for a dramatically lower contribution rate.

The choice of adjustment parameters β and γ must navigate an intertemporal policy tradeoff. Contributions need to rise in the short run to accumulate the assets required for the long-term reduction in c^*. Thus, the tradeoff is between speed of reaching c^* vs. tempering the short-term rise in c required to reach a^*. Suppose we set a target of approaching c^* by year 30 (corresponding to a somewhat conventional time horizon for actuarial amortization schedules) and set the contribution adjustment parameter β equal to 0.5 (half speed). Then we find that the tradeoffs are plausibly managed by choosing the asset adjustment parameter γ near the maximum value for monotonic convergence, $\gamma_{m/o} = 0.075$, on the blue curve in Figure 4.

Figure 5 depicts the corresponding paths for the contribution rate (red curve, on the right scale) and asset ratio (blue curve, on the left scale). This path raises the contribution rate for about 7 years to a maximum of 36 percent (a 9-point hike), before ultimately dropping down to approximately 10 percent by year 30. Setting β any faster requires a sharper short-term rise in contributions and setting it any slower fails to approach c^* that closely in 30 years.

Our dynamic analysis shows how to generate a smooth adjustment path to "full" funding, unlike the actuarial scenario of the contribution cliff envisioned upon completion of a closed interval amortization schedule.[17] The path depicted is challenging: it calls for a substantial rise in contributions over the near term and the potential long-term gain is by no means certain. As noted previously, c^* is highly sensitive to the assumed return. At $R = 1.05$, $c^* = 0.24$, in which case the accumulation of assets requires a much larger hike in short-run contributions (over 20 points), for very little gain in the long-run. Moreover, even if the assumed return is accurate in expected value, the distribution of outcomes can be very wide under stochastic returns.

Stochastic Simulations

Let us consider a stochastic model of the path to "full" funding. We ran Monte Carlo simulations of the adjustment path, with R distributed as lognormal, mean 1.07 and standard deviation of 0.15.[18] Figure 6 depicts the trajectories for the contribution rate and asset ratio at the median, 25th, and 75th percentiles of their distributions.

[17] Open interval amortization has no such cliff, but often never approaches the funding target. (See, for example, Costrell 2018b, Figure 3.)

[18] We estimated the standard deviation values associated with specific target returns using the publicly available, forward looking capital market assumptions published by Callan in early 2020 (pre-pandemic). We estimated the portfolio allocation that would generate each target return across a diversified portfolio including large cap U.S. equities (e.g., S&P 500), small/mid Cap U.S. equities (e.g., Russell 2500), Global ex-U.S. Equity (e.g., MSCI ACWI ex USA), real estate (e.g., NCREIF ODCE), private equity (e.g., Cambridge Private Equity), and aggregate U.S. bonds (e.g., Bloomberg Barclays Aggregate). We then applied that allocation using Callan's estimated standard deviation and asset class correlations to calculate the associated standard deviation values for each return.

The first point to note is that the median of these distributions is indistinguishable from the deterministic trajectories depicted in Figure 5. The mean (not depicted) differs, due to the asymmetry of the lognormal distribution, but would also track the deterministic case if the distribution were symmetric; this gives us some basis for interpreting the steady-state values we derived analytically as expected values, which will be useful in the next section.

That said, Figure 6 clearly shows the huge risk in these trajectories, as illustrated by the spread between the 25^{th} and 75^{th} percentiles. These risks are unavoidable, with investment in risky assets. Moreover, the risk rises (the spread widens) over time, contrary to the popular notion that the good years and the bad years "average out" over time.[19] Given the assumed asset allocation, the only latitude in managing that risk is the degree to which it falls on the contribution rate or the asset ratio. Under the adjustment parameters depicted, which seemed reasonable for the deterministic case ($\beta = 0.5$, $\gamma = 0.075$), quite a bit of the risk falls on the contribution rate: the 25-75 percentile spread widens to over 50 percentage points by year 30.[20]

The contribution risk can be reduced, pushing it instead onto asset risk, by dampening the adjustment parameters. For example, if we cut γ in half to 0.0375, choosing a slower trajectory toward the targets of $a^* = 7$ and $c^* = 0.10$, we find a somewhat smaller contribution risk, as depicted in Figure 7. The 25-75 percentile spread is narrower than in Figure 6 (about 35 percentage points, instead of 50) and the spread for the asset ratio (not shown, for purposes of clarity) is somewhat wider. That said, as in the previous case, the 25-percentile asset ratio never dips as low as 4, and the risk of insolvency is negligible, unlike policy simulations with a constant contribution rate in papers discussed below.

[19] The law of large numbers applies to the annual rate of return, but not to the total return, or the assets on which the return is earned. Figures 6 and 7 visually illustrate the Fallacy of Time Diversification.
[20] The high probability of contributions going negative represents the chance of a run of good returns leading to asset accumulation far beyond the target, so excess assets are drawn down to pay benefits.

To reiterate, these simulations are not meant to be policy prescriptions, but rather to illustrate how contribution policy might be reformulated, using adjustment parameters toward asset and contribution targets. We believe this approach holds promise, compared to the current actuarial approach, even as it would require further refinement. Such a policy would not mitigate risk – only a change in asset allocation would do that (as discussed below) – but illustrates how it can be deliberatively used to apportion risk between assets and contributions.

It is, perhaps, noteworthy that the approach set out here is consistent with a formal result from stochastic control theory. In a model with one state variable and one control variable (here, the asset ratio and contribution rate, respectively), and a quadratic loss function in the two variables, the optimal control is of the type we have considered, linear in the two gaps. Moreover, as shown by Turnovsky (1974), the introduction of stochastic elements dampens the adjustment in the optimal control, resulting in slower response in the target variable, consistent with the example we have illustrated here in Figure 7 vs. Figure 5.

Finally, whether the model is deterministic or stochastic, one needs an anchor for the asset accumulation goal. That anchor has traditionally been based on liabilities, and reasonably so, but, as we have argued above, the target should be based on true liabilities, appropriately discounted. Whether that target should be 100 percent of true liabilities or 60 percent, as in the $a^* = 7$ simulations depicted (corresponding to 100 percent of actuarial liabilities) or something less, and how to approach that question, is the subject to which we now turn. Our goal is to integrate the insights from our previous analysis of the steady-state (expected) contribution rate, and the risk thereof, as a function of the target funded ratio.

IV. CHOOSING THE TARGET FUNDED RATIO: A SIMPLE OPTIMIZATION FRAMEWORK

The question of whether liabilities should be fully funded has long been the subject of economic analyses focusing on different aspects of public pension finance.[21] This literature, of course, begins with Samuelson's seminal analysis (1958), where the optimality of pre-funding vs. pay-go turns on whether the rate of return exceeds the growth rate.

Bohn (2011) constructs a model based on the higher intermediation costs faced by individual borrowers than public entities, so that it is efficient for public pension plans to take on debt on behalf of the taxpayers. As a result, the optimal funding level is less than 100 percent and may well be zero (pay-go). As Bohn (2011) notes, a countervailing feature is that unfunded pensions cannot be fully guaranteed, and this would lead employees to require a wage premium for that risk; pre-funding, then, may be rational for employers, since pension assets can serve as collateral to spare taxpayers the cost of that premium.

Various other aspects of political economy (transparency, agency problems, distorted political time horizons) may also argue for pre-funding, to one extent or another (Brown, Clark, and Rauh (2011, section 3), Glaeser and Ponzetto (2014)). In addition, the traditional interpretation of intergenerational equity holds that each generation of taxpayers should pay for its own full cost of public services, including pre-funding benefits.

More recently, Lenney, Lutz, Schule, and Sheiner (2021) challenge this latter view, observing that rising contributions to pay down pension debt have burdened current generations with the cost of benefits for prior generations, instead of spreading these costs over the indefinite future. Their recommended funding policy is, instead, to simply maintain current pension debt ratios (appropriately discounted), a policy that effectively takes the current true funded ratio as

[21] For a brief survey of such work, see Brown, Clark, and Rauh (2011, section 3), in their introduction to the special issue of *JPEF* on the research commissioned for an NBER project on the economic analysis of public pensions.

given and sets the steady-state contribution rate based on that status quo. That contribution rate is governed by the expected rate of return; under their preferred scenarios of modest returns, aggregate contribution rates would only need a moderate hike to stabilize debt ratios.

Lucas (2021) and Rauh (2021), point out the difficulties that arise from using a risky rate of return in a deterministic model, even as liabilities are discounted at a risk-free rate. As a result, Rauh (2021) argues the debt-stabilizing contribution rate may well rise much higher than Lenney, Lutz, Schule, and Sheiner (2021) suggest. He also points out that their paper assumes away the cost of insolvency in their stochastic simulations by unrealistically assuming plans can borrow through periods of negative assets. As he points out, rating agencies factor in the risk of insolvency, and, hence, the risk that contributions would jump to the pay-go rate. Lucas (2021) also places great emphasis on the risk of insolvency, arguing that the goal of (expected) debt-stabilization is a less suitable definition of sustainability than insolvency-minimization.

A question that arises in tandem with the optimal funded ratio is that of the optimal asset allocation – how much risk the government sponsor, and thus taxpayers, should bear.[22] While a traditional finance approach would match risk-free income streams to the stream of promised payments, the case has been made for some equity holdings to hedge against wage fluctuations correlated with equity prices.

Pennacchi and Rastad (2011) formally analyze the pension portfolio risk that maximizes utility of a representative taxpayer. Factors in favor of some public risk-bearing include superior access to risky investments, analogous to Bohn's (2011) consideration of lower intermediation costs. Similarly, factors against risky portfolios include the agency problem that upside outcomes may not fully benefit taxpayers but may instead lead to enhanced employee benefits.

[22] Here, too, we draw on the brief survey of such work in Brown, Clark, and Rauh (2011, section 3).

A Simplified Social Welfare Function and Contribution Rate Risk

The approach we sketch out here contributes to this literature by integrating our steady-state analysis into a very simple (and, hence, only semi-formal) social welfare function. We abstract from the specific features highlighted in the literature above, collapsing all such considerations into a setup that represents the two main tradeoffs: short-run vs. long-run costs; and expected contributions vs. risk, as valued by the public and/or policymakers (depending on one's interpretation). We analyze the joint optimization of the funded ratio and the risk profile of the asset allocation. Using our results from *(3*)* in this framework allows us to present the marginal benefits and costs of these choices in readily understood terms and helps clarify the logic of pre-funding, incorporating the public's (or policymaker's) tolerance for risk.

Specifically, our optimization analysis exploits the insights from our steady-state result that distinguishes between r and d. In so doing, we arguably help resolve a bit of schizophrenia in the debate over actuarial discount rates. It is increasingly (if grudgingly) recognized, even by non-economists, that liabilities should be discounted at a low-risk rate corresponding to the guarantee of promised benefits. Yet finance economists typically restrict their conclusion to reporting requirements, and not necessarily to funding policy. Our approach integrates dual rates – specifically, the risk premium $(r - d)$ – into a framework for funding policy that simultaneously represents the benefits and costs of funding pension payments from risky assets.

Here is our framework. In general terms, the risk profile and target funded ratio should be based on the public's preferences for intergenerational cost sharing and its tolerance for risk in pursuit of returns. Let us posit an (over-) simplified social welfare function:

$$-V[(a^* - a_0), E(c^*), \sigma(c^*)],$$

where $(a^* – a_0)$ is a shorthand measure of the costs required (non-discounted) over some period to reach the asset target; $E(c^*)$ is the expected value of the steady-state contribution rate at the asset target; and $\sigma(c^*)$ is the risk of c^* from relying on asset income. Since these arguments to V are social "bads," we preface V with a minus sign,[23] so the partials V_1, V_2, and V_3 are positive.

More precisely, let us think of c^* as the contribution rate c_t as t gets large (e.g., t = 30), by which point $E(c_t)$ approaches the steady-state value $E(c^*)$ given by (3*). Similarly, we can think of $\sigma(c^*)$ as the corresponding contribution risk, $\sigma(c_t)$, but it is not a steady-state value, since risk continually rises, as discussed above and shown below. We may instead think of it as the contribution risk as the expected contribution rate approaches its steady-state value.

Specifically, for analytical purposes we posit:

(5) $\sigma(c_t) = s(\sigma(r); t, \beta, \gamma)f^*\lambda^* \equiv S(r; t, \beta, \gamma)f^*\lambda^*.$

This formulation lets the contribution risk rise with time, as illustrated in our simulations below. At any given time (e.g., t = 30), we express the contribution risk as a general function s of the annual risk of r, $\sigma(r)$, which itself is a function of the annual risk premium, $(r – d)$, where d is considered a parameter. As we shall see, the shape of the composite function $S(r; t, \beta, \gamma) \equiv s(\sigma(r); t, \beta, \gamma)$ will be important below but is left unspecified here.

The one substantive assumption in (5) is that the long-term contribution risk is multiplicative in the target asset ratio $a^* = f^*\lambda^*$. This assumption is analytically convenient, as it parallels the fashion in which $a^* = f^*\lambda^*$ enters $E(c^*)$ in (3*). Fortunately, this assumption appears to be a very close approximation for our simulations of $\sigma(c_{30})$ – which we take to illustrate $\sigma(c^*)$ – as we vary a^* in the relevant range.[24] The key take-away from this formulation

[23] Equivalently, we could cast the problem as minimizing social dis-welfare, $V[(a^* - a_0), E(c^*), \sigma(c^*)]$.
[24] Specifically, we find that as we raise a^* from 7 to 9, the ratio of $\sigma(c_{30})$ to a^* varies by less than one percent.

is the parallel role of $(r - d)$ in $E(c^*)$ and $\sigma(c^*)$ in (3*) and (5) (via $\sigma(r)$). Thus, we simultaneously represent both the benefit and cost of the chosen degree of risk.

Optimal Risk Profile

The optimization problem requires a joint decision on two instruments: (i) the risk profile of the asset allocation, formally represented by the target return r (for given d); and (ii) the target funded ratio, f^*. Taking these in turn, we first optimize $-V[(a^* - a_0), E(c^*), \sigma(c^*)]$ over r, conditional on the funding target f^*. The choice variable r enters $E(c^*)$ and $\sigma(c^*)$ through the risk premium, $(r - d)$. Thus, from (3*), we have $dE(c^*)/dr = -f^*\lambda^*$, and from (5), we have $d\sigma(c^*)/dr = S'(r)f^*\lambda^*$, where, for simplicity, we omit the givens, (t, β, γ).

Consequently,

$$(6)\ - dV[(a^* - a_0), E(c^*), \sigma(c^*)]/dr = - V_2\, dE(c^*)/dr - V_3\, d\sigma(c^*)/dr$$

$$= [V_2 - V_3 S'(r)]f^*\lambda^*.$$

The bracketed term simply represents the balance of social weights between additional risk and return. We assume there is an interior optimum for (5) with $r > d$,[25] where $V_2 = V_3S'(r)$.

Figures 8A and 8B illustrate the tradeoff between the contribution rate and risk for selected target rates of return. These simulations are similar to those above but depict risk with the standard deviation of the contribution rate over time, rather than the 25-75 spread. As above, we use the Callan (2020) capital market assumptions and vary the composition of the diversified portfolio to obtain the target returns depicted. These assumptions generate $\sigma(r)$, which feed into the simulations that generate the outcomes for c_t, $E(c_t)$ and $\sigma(c_t)$.

[25] This would hold, for example, with a quadratic social welfare function.

Figure 8A depicts the trajectories for the median contribution rate, as r varies from 4 percent to 7 percent.[26] The lower trajectories for higher target returns illustrate the benefit from more aggressive asset allocations. Figure 8B depicts trajectories for the standard deviation of the contribution rate, as r varies.[27] The higher trajectories for higher target returns illustrate the cost from riskier asset allocations. Moreover, the rate at which the risk rises (the increasing gaps between the curves) exceeds that of the benefit (the drop between the curves in Figure 8A). That is, the "price" of seeking higher returns rises as the plan gets more aggressive.[28] This corresponds to the convexity of the composite function $S(r)$ in (5), as will be important below.

Optimal Target Funded Ratio

We now turn to our main focus, the optimization of $-V[(a^* - a_0), E(c^*), \sigma(c^*)]$ over the choice variable f^*. We consider the marginal social impact – benefit or cost – of raising the funded ratio. Qualitatively, the benefit is the reduction in the long-run expected contribution rate, $E(c^*)$, and the two costs are the short-run rise in contributions to accumulate more assets, $(a^* - a_0)$, and the increased risk of c^*, from raising the portion of benefit payments defrayed by risky investment income.

Formally, we consider the three pieces of

$$(7)\quad -dV[(a^* - a_0), E(c^*), \sigma(c^*)]/df^* = -V_1\, da^*/df^* - V_2\, dE(c^*)/df^* - V_3\, d\sigma(c^*)/df^*.$$

The first piece is the marginal social cost in the short run to accumulate more assets, $a^* = f^*\lambda^*$:

$$(8)\quad -V_1\, da^*/df^* = -V_1\, \lambda^*.$$

[26] As noted earlier, the mean contribution rate is lower due to the asymmetry of the distribution of r under the lognormal assumption. This deviation between mean and median contribution rate is fairly minimal by year 30 for r = 4% and 5%. It widens notably by year 30 for r = 6% and quite substantially for r = 7%.

[27] Note that the low-risk portfolio, r = 4%, is not risk-free. That is because the fixed rate bond portfolio (r = 2.75%) is not risk-free ($\sigma(r)$ = 3.75%) and, to reach the target return of 4%, one must add an equity component. Thus, although our simulations illustrate the analysis below, they do not accord exactly with the risk-free assumption.

[28] The change in risk per unit change in median contribution rises from 0.9 to 1.4 to 2.6 as r varies from 4% to 7%.

The second piece is the marginal social benefit of reducing the expected long-run contribution rate, $E(c^*)$, by raising f^*. From (3*), we have:

$$(9)\ \ – V_2\, dE(c^*)/df^* = V_2\, [(c^p − c^n) + (r − d)\lambda^*].$$

Note that the magnitude of this benefit depends on how aggressive the asset allocation is, $(r – d)$.

Taking these first two pieces of (7) together and using (2*) for λ^*, we find the net social benefit (ignoring the cost of risk for the moment) of raising the target funded ratio is positive if $V_2\, (r – g) > V_1$. This condition is just a simplified version of the usual intergenerational tradeoff. Suppose, to take the simplest example, the accumulation of additional assets is immediate. The subsequent reduction in the contribution rate, as a percent of payroll, represents a perpetuity that grows at rate g. Then, if social cost is simply the present value of contributions, current and future, discounted at a social discount rate δ, we find $V_1 = 1$ and $V_2 = 1/(\delta − g)$. Consequently, the net benefit is positive if and only if $(r – g)/(\delta − g) > 1$, i.e., $\delta < r$, a standard result.[29]

Finally, the third term in (7) is the marginal social cost of the increased risk from relying on the additional income generated by asset accumulation. Using (5) for $\sigma(c^*)$, we have:

$$(10)\ – V_3\, d\sigma(c^*)/df^* = – V_3 S(r)\lambda^*.$$

Pulling these three pieces together, we have:

$$(7')\ – dV[(a^* - a_0), E(c^*), \sigma(c^*)]/df^*$$

$$= – V_1\,\lambda^* + V_2\, [(c^p − c^n) + (r − d)\lambda^*] – V_3 S(r)\lambda^*.$$

As discussed earlier, the risk premium, $(r − d)$, simultaneously conveys the benefit and the cost of risky investment. These are reflected in the second and third terms above, respectively, since $S(r) \equiv s(\sigma(r))$ and $\sigma(r)$ is a function of $(r − d)$.

[29] Adding convexity to the annual disutility of contributions is straightforward.

In the second term above, it is important to reiterate that c^n is discounted at d, not r. Thus, the gap between c^p and c^n is much narrower than under actuarial accounting. The benefit from asset accumulation is smaller in that regard but enhanced by the risk premium.

We can further clarify how the costs and benefits of risky investment factor into the optimality condition for f^* by regrouping terms in *(7')* and substituting from (6) the optimality condition for the target return, $V_2 = V_3 S'(r)$:

$$(7'') \quad -dV[(a^* - a_0), E(c^*), \sigma(c^*)]/df^*$$

$$= -V_1\lambda^* + V_2(c^p - c^n) + [V_2(r-d) - V_3 S(r)]\lambda^*.$$

$$= -V_1\lambda^* + V_2(c^p - c^n) + V_2[(r-d) - S(r)/S'(r)]\lambda^*.$$

Our analysis assumes d is risk-free, so $S(r{=}d) = s(\sigma(r{=}d)) = 0$. Thus, $S'(r) > S(r)/(r-d)$, if $S(r)$ is convex, as seems likely (to be explained below). This means the bracketed term is positive: the marginal benefit from reducing contributions by the extra income from additional risky assets outweighs the cost of the extra risk.

Convexity of the composite function $S(r) = s(\sigma(r))$ depends on the convexity of both $s(\sigma)$ and $\sigma(r)$, where, to recall, $s(\sigma)$ relates the risk of the long-term contribution rate to that of the annual return and $\sigma(r)$ gives the risk of the annual return as a function of its expected value. Under basic portfolio theory, where assets are simply a mix of the risk-free asset and the market portfolio, $\sigma(r)$ is linear. Under the more complex capital market assumptions we use for our simulations, $\sigma(r)$ embeds some convexity, but, as shown in Figure 9, that convexity is apparently swamped by the convexity of $s(\sigma)$.

As Figure 9 shows, $S(r)$ is notably convex for $t = 30$ (i.e., $\sigma(c_{30})$). This contrasts with the near-linear appearance of $\sigma(r)$, superimposed on the same graph. Thus, we infer that the lion's share of $S(r)$'s convexity is due to that of $s(\sigma)$, the non-linear impact on the contribution risk of the risk in annual returns, rather than the non-linearity of the annual risk-return relationship. This non-linearity is apparently due to compounding of risk over time, as illustrated by the comparison between the curvature of $S(r)$ for $t = 30$ and $t = 15$.

To summarize, at the joint optimum the marginal social cost of accumulating more assets (the first term in *(7''))* is balanced by two marginal benefits (the second and third terms in *(7''))*. The reduction in steady-state contributions from pre-funding instead of pay-go is $(c^p - c^n)$; and the net benefit from the additional income from risky assets is $[(r - d) - S(r)/S'(r)]\lambda^*$.

We can gain some purchase on the potential relative magnitude of these two benefits by further analysis of these two terms. Substituting from (2*) for λ^*, and rearranging, we find:

$$(11)\quad (c^p - c^n) + [(r - d) - S(r)/S'(r)]\lambda^* = (c^p - c^n)\{1 + [1 - (S(r)/(r - d))/ S'(r)](r - d)/(d - g)\}$$

$$= (c^p - c^n)\{1 + [1 - \mathit{arc\ slope/tangent}](r - d)/(d - g)\},$$

where $(1 - \mathit{arc\ slope/tangent})$ is a measure of the convexity of $S(r)$ at r, over the interval (d, r).

To illustrate magnitudes, let $d = 0.04$, and $g = 0.03$, as before, and consider the *arc slope* and *tangent* from Figure 9. For $r = 0.07$, we estimate *arc slope/tangent* at about 0.44, which implies that the net benefit from risky investments is about 1.67 times the straight pre-funding benefit, $(c^p - c^n)$. At $r = 0.06$, we estimate that multiple drops to about 1.09 times $(c^p - c^n)$, thus doubling the straight pre-funding benefit, and at $r = 0.05$, that multiple drops to about 0.48. These estimates are by no means dispositive; they merely illustrate the potential relative size of the two net marginal benefits from accumulating more assets.

This exercise leads us to re-examine the basis for pre-funding in light of the dual rates. By properly discounting liabilities at d instead of r, the normal cost rate is dramatically elevated, and $(c^p - c^n)$ is greatly diminished, as we have replaced the gap between r and g with the much narrower gap between d and g.[30] It is the former gap that is often taken as the Samuelsonian case for pre-funding. That is also what underlies the actuarial goal of "full" funding and contributing in steady state at the normal cost rate (discounted at r) instead of the much higher pay-go rate.

Our analysis suggests that the much narrower gap between c^p and c^n (discounted at d) might, in fact, be the less powerful motive for pre-funding than the exploitation of risky investments. We should emphasize, however, that the strength of this latter motive may reflect excessive risk-tolerance on the part of policymakers. If so, we would have the ironic result stated in the introduction that the case for pre-funding – considered the course of prudence – may rest largely on imprudent attitudes toward risk.

Finally, we should also emphasize that the object of our analysis, the optimal funded ratio f^*, is applied to a much higher liability ratio, λ^*, discounted at d. Thus, even if the above analysis is taken to suggest a reduction in f^*, it does not resolve the issue of whether current funding targets (100 percent of reported liabilities or 60 percent of true liabilities) are too high or too low. A low target funded rate for true liabilities may well exceed current funding targets.

V. CONCLUSIONS AND POLICY GUIDANCE

Standard actuarial practice pursues intergenerational equity and sustainability by employing funding rules that seek to ensure each generation pays for the services it receives. These rules operate through the concepts of normal cost and amortization, which, together, aim to fully fund benefits for each cohort of workers and taxpayers. Normal cost is meant to pre-fund

[30] In the limit, as $d \to g$, $c^n \to c^p$, so the second term in $(7'')$, $V_2(c^p - c^n)$, vanishes. By L'Hôpital's rule, as $d \to g$, $\lambda^* = (c^p - c^n)/(d - g) \to -c^{n\prime}(d)$, so it does not vanish from the first term of $(7'')$. Thus, in this limiting case, as $d \to g$, there will be no interior solution for f^* in the absence of the third term. Lenney, Lutz, Schule, and Sheiner (2021 Table A9) put rediscounted normal cost higher than the pay-go rate, apparently assuming $d < g$.

the full cost of benefits earned by a cohort of employees over their careers, while amortization is meant to close funding gaps that result from payment shortfalls and unrealized assumptions.

In practice, these rules have failed to achieve intergenerational equity or sustainability. The true market cost of earned benefits have been understated, leading to the accumulation of large pension debt and steeply rising contributions to amortize that debt. These payments are crowding out spending in other areas like infrastructure and education, as current generations are paying for past benefits. Going forward, neither intergenerational equity nor sustainability are embedded in a deliberative policy choice framework that adequately considers the risks involved for future generations of public workers and taxpayers.

The shortcomings of the actuarial approach lie not only in practice, but in theory, as discounting by the expected return fails to convey the cost of risk. The reformulation we propose goes back to fundamentals, properly discounting liabilities and incorporating the portfolio's risk premium to simultaneously represent the benefits and costs of risky investment. Our approach can be thought of as replacing both pieces of actuarial contributions: normal cost and amortization, in a way that explicitly recognizes risk.

Normal cost is effectively replaced by the steady-state expected contribution rate, derived from the laws of motion for assets and liabilities and a target funded ratio, chosen by policymakers, but honestly measured with a low-risk discount rate. As a result, the steady-state contribution rate: (i) broadens the normal cost piece (wrongly discounted) to a blend of normal cost (properly discounted) and pay-go, weighted by the target funded ratio; and (ii) folds in the benefit of excess returns $(r - d)$ in exchange for the risk borne by the sponsoring government.

Instead of an amortization schedule, our approach specifies an adjustment process to the steady-state contribution rate and target asset ratio. We have shown how to set the adjustment parameters, starting with the deterministic case. However, as also show, in a stochastic world, the risk widens over time, even as the expected contribution rate approaches a steady state. We illustrate how the adjustment parameters might be modified from the deterministic case to better manage risk. That choice of parameters must navigate the tradeoff between lower contribution risk and speed of adjustment toward the targets.

Finally, we sketch out a social welfare framework for choosing a target funded ratio, f^*, as applied to true liabilities (discounted at d), and a target expected return on assets, r, with its associated risk. This simple framework balances intergenerational equity, the quest for returns, and investment risk based on the sponsoring government's assessment of public preferences. Incorporating our steady-state results into this simple framework sheds new insight on the costs and benefits of asset accumulation. The standard rationale for pre-funding vs. pay-go ($c^n < c^p$), is attenuated by properly discounting normal costs but is augmented by the net social benefit of the excess returns from risky investments.

At the optimal asset allocation, we show that the net benefit of those excess returns depends on how aggressive the policymakers are with respect to risk and the convexity of long-run risk with respect to the expected return. Our simulations of long-run risk show notable convexity, which appears to compound over time the very modest annual convexity in the risk of annual return. As a result, this second rationale for pre-funding – the net social benefit of excess returns – can be quite substantial and even outweigh the traditional rationale for pre-funding.

To be sure, the optimization framework we present is nice, but clearly not descriptive of current practice in many respects. Indeed, under actuarial practice, the asset target decision (a^*) does not appear to be based on any optimization framework, implicit or explicit. Rather, a^* is simply set at 100% of the actuarial calculation of liabilities, corresponding to about 60% of true liabilities. Given the latitude plans seem to exercise in choosing the discount rate and other assumptions, one might interpret these decisions as, in effect, reverse engineering asset and contribution targets to satisfy policymaker preferences over some heuristic objective function.

To speculate along these lines, we could characterize common critiques of pension funding policy as: (i) understating V_2 relative to V_1 – excess time preference; (ii) under-estimating the social cost of risk, V_3 relative to V_2 – insufficient risk aversion; or (iii) underestimating the amount of risk, $\sigma(c^*)$, perhaps due to misplaced confidence in time diversification, excessive self-confidence in investment acumen based on past good luck (Andonov and Rauh, 2022), and/or the distorted incentives from U.S. public pension accounting rules (Andonov, Bauer, and Cremers, 2017).

Would a proper evaluation of costs and benefits lead us to raise or reduce the target asset accumulation and contribution rate? Examining *(7')* we see that excessive time preference reduces the second term and insufficient risk aversion raises the third term. This leads us to an inconclusive assessment of whether the (true) target funded ratio is too high or too low, even before getting into the weeds of convexity and the like.

What about the steady-state expected contribution rate? What does our analysis say (if anything) about how it compares with the actuarial steady state, namely the wrongly discounted normal cost rate? Our discussion of (3*) was inconclusive on the impact of proper discounting on c^*, but our calculations based on (1*) might arguably be more informative in this regard. If the asset target $a^* = 7$ is about right, then the steady-state contribution rate of about 10 percent, depicted in Figures 5 and 6 is in the same ballpark as the reported normal cost rate of about 13 percent, depicted in Figure 1. Of course, that rests heavily on the accuracy of the expected return at 7 percent; at $r = 6$ percent, $c^* = 17$ percent and at $r = 5\%$, $c^* = 24$ percent.

In any case, the prospect of any such relief from the current rate of 27 percent is small consolation, given the transition paths to a^* = 7 depicted in these figures, even for r = 7 percent. These paths exhibit substantial short-term hikes, dramatically widening risk, and quite possibly never declining at all. Choosing a less aggressive portfolio and judicious adjustment parameters reduces the risk, but also reduces the prospect of substantial long-run decline in contributions.

In short, there are no good choices, but there may be better and worse ones. Our hope is that the analysis provided here helps elucidate the tradeoffs involved in pursuit of pension funding sustainability and intergenerational equity, and how these tradeoffs might inform the approach to contribution policy we propose. It is an approach that integrates proper liability discounting with clearer consideration of the benefits and costs of risks in setting asset and contribution targets, while pointing the way to a deliberative process of adjustment toward those targets, while managing the risks better than current policies.

Appendix: Convergence Conditions

We can usefully express the system (1″) and (4) in matrix form:

$$\begin{bmatrix} a \\ c \end{bmatrix}_{t+1} = \begin{bmatrix} (R/G) & (1/G) \\ -\gamma & (1-\beta) \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix}_t + \begin{bmatrix} (-c^p/G) \\ (\gamma a^* + \beta c^*) \end{bmatrix}.$$

Denote the transition matrix above by A. The asymptotic stability condition (see Neusser (2021), equation (3.18), p. 84) is: $|tr\,(A)| < 1 + det(A) < 2$. In the present case, this implies

(i) $\gamma > \beta(R - G) \equiv \gamma_{min} > 0$, and

(ii) $\gamma < G - R(1 - \beta) \equiv \gamma_{max}$.

The condition for asymptotic oscillation is $[tr\,(A)]^2 < 4 \cdot det(A)$, or, in the present case:

(iii) $\gamma > G[(R/G) - (1 - \beta)]^2/4 \equiv \gamma_{m/o}$.

References

Andonov, Aleksandar, Rob M. M. J. Bauer, and K. J. Martjin Cremers, 2017. "Pension Fund Asset Allocation and Liability Discount Rates," The Review of Financial Studies, 30(8), 2555-2595.

Andonov, Aleksandar, and Joshua D. Rauh, 2022. "The Return Expectations of Public Pension Funds," *The Review of Financial Studies*, 35(8), 3777-3822.

Biggs, A. (2011). An Options Pricing Method for Calculating the Market Price of Public Sector Pension Liabilities. *Public Budgeting & Finance*, 31(3), 94-118.

Bohn, Henning, 2011. "Should Public Retirement Plans Be Fully Funded?" *Journal of Pension Economics and Finance*, Vol. 10, no. 2, pp. 195-219.

Brown, Jeffrey R., Robert Clark, and Joshua Rauh, 2011. "The Economics of State and Local Pensions," *Journal of Pension Economics and Finance*, Vol. 10, no. 2, pp. 161-172.

Brown, J., & Wilcox, D. (2009). Discounting State and Local Pension Liabilities. *American Economic Review*, 99(2), 538-42.

Costrell, Robert M., 2018a. "The 80 Percent Pension Funding Target, High Assumed Returns, and Generational Inequity," *Contemporary Economic Policy*, Vol. 36, no. 3, pp. 493-504.

Costrell, Robert M., 2018b. "Accounting for the Rise in Unfunded Public Pension Liabilities: Faulty Counterfactuals and the Allure of Simple Gain/Loss Summations," *Journal of Pension Economics and Finance*, January 2018 (vol. 17, no. 1), pp. 23-45.

Costrell, Robert M. and Josh McGee, 2020. "Sins of the Past, Present, and Future: Alternative Pension Funding Policies," EdWorkingPaper: 20-272, Annenberg Institute at Brown University: August 7, 2020; conference paper for Brookings Municipal Finance Conference, July 13-14, 2020.

Glaeser, Edward L., and Giacomo A.M. Ponzetto. 2014. Shrouded Costs of Government: The Political Economy of State and Local Public Pensions. *Journal of Public Economics* 116: 89-105.

Lenney, Jamie, Byron Lutz, and Louise Sheiner, 2019a. "The Sustainability of State and Local Government Pensions: A Public Finance Approach," Brookings Municipal Finance Conference; 2019b; University of Chicago Policy Forum: the Pension Crisis.

Lenney, Jamie, Byron Lutz, Finn Schule, and Louise Sheiner, 2021. "The Sustainability of State and Local Pensions: A Public Finance Approach," *Brookings Papers on Economic Activity*, Spring, pp. 1-48. Online Appendix.

Lucas, Deborah, 2021. "The Sustainability of State and Local Pensions: A Public Finance Approach, Comment" *Brookings Papers on Economic Activity*, Spring, pp. 49-57.

Neusser, Klaus, 2021. Difference Equations for Economists.

Novy-Marx, R., & Rauh, J. D. (2009). The Liabilities and Risks of State-Sponsored Pension Plans. *Journal of Economic Perspectives*, 23(4), 191-210.

Novy-Marx, Robert, and Joshua D. Rauh, 2011. "Public Pension Promises: How Big Are They and What Are They Worth?" *Journal of Finance*, 66(4), 2011, 1211-1249.

Pennacchi, George and Mahdi Rastad, 2011. "Portfolio allocation for public pension funds." *Journal of Pension Economics and Finance*, Vol. 10, no. 2, pp. 221-245.

Rauh, Joshua, 2021. "The Sustainability of State and Local Pensions: A Public Finance Approach, Comment" *Brookings Papers on Economic Activity*, Spring, pp. 57-61.

Samuelson, Paul, 1958. "An exact consumption loan model of interest with or without the social contrivance of money," *Journal of Political Economy*, December, 66 (6), pp. 467-482.

Turnovsky, Stephen J., 1974, "The Stability Properties of Optimal Economic Policies," *American Economic Review*, March, 64 (1), pp. 136-148.

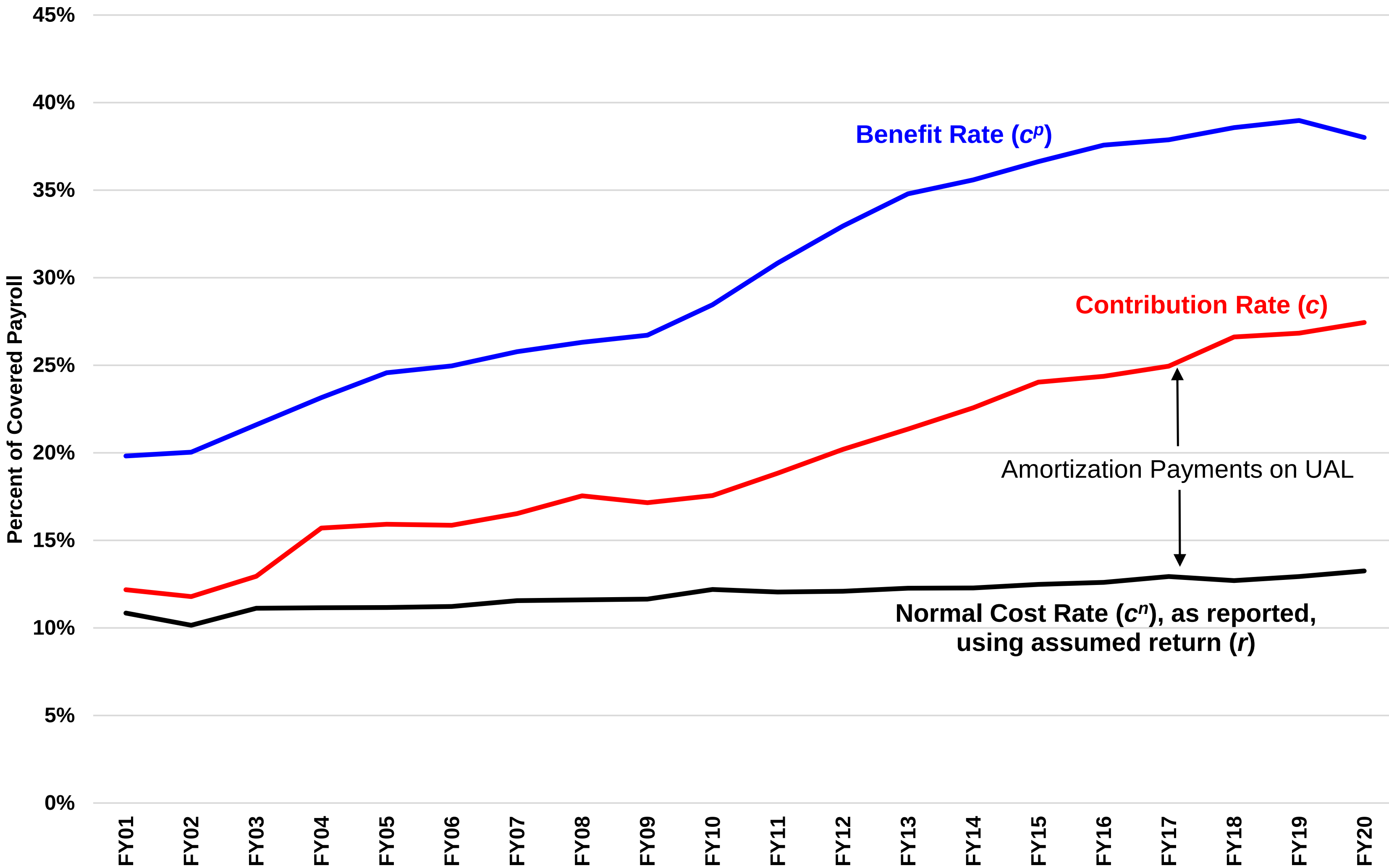
Table 1: Pension Funding Notation
A = assets on hand
L = accrued liabilities, the present value of future benefits earned to date
f = funded ratio, A/L (full funding goal is f = 100%)
W = payroll
$a = A/W$ = assets/payroll
$\lambda = L/W$ = liabilities/payroll
c = contribution rate, % of payroll
c^p = benefit payments as % of payroll ("pay-go rate")
c^n = newly accrued liabilities as % of payroll ("normal cost rate")
r = return on assets; $R = (1+r)$
d = discount rate used to calculate present value of liabilities; $D = (1+d)$
g = growth rate of payroll; $G = (1+g)$

Table 2: Convergence Conditions for Adjustment Parameters

$$c_{t+1} = c_t + \beta(c^* - c_t) + \gamma(a^* - a_t)$$

Range of γ	**Asymptotic Behavior of *(1″)-(4)***
$\gamma < \gamma_{min} \equiv \beta(R - G)$	Monotonic divergence
$\gamma_{min} < \gamma < \gamma_{m/o} \equiv G[(R/G) - (1 - \beta)]^2/4$	Monotonic convergence
$\gamma_{m/o} < \gamma < \gamma_{max} \equiv G - R(1 - \beta)$	Oscillatory convergence
$\gamma_{max} < \gamma$	Oscillatory divergence

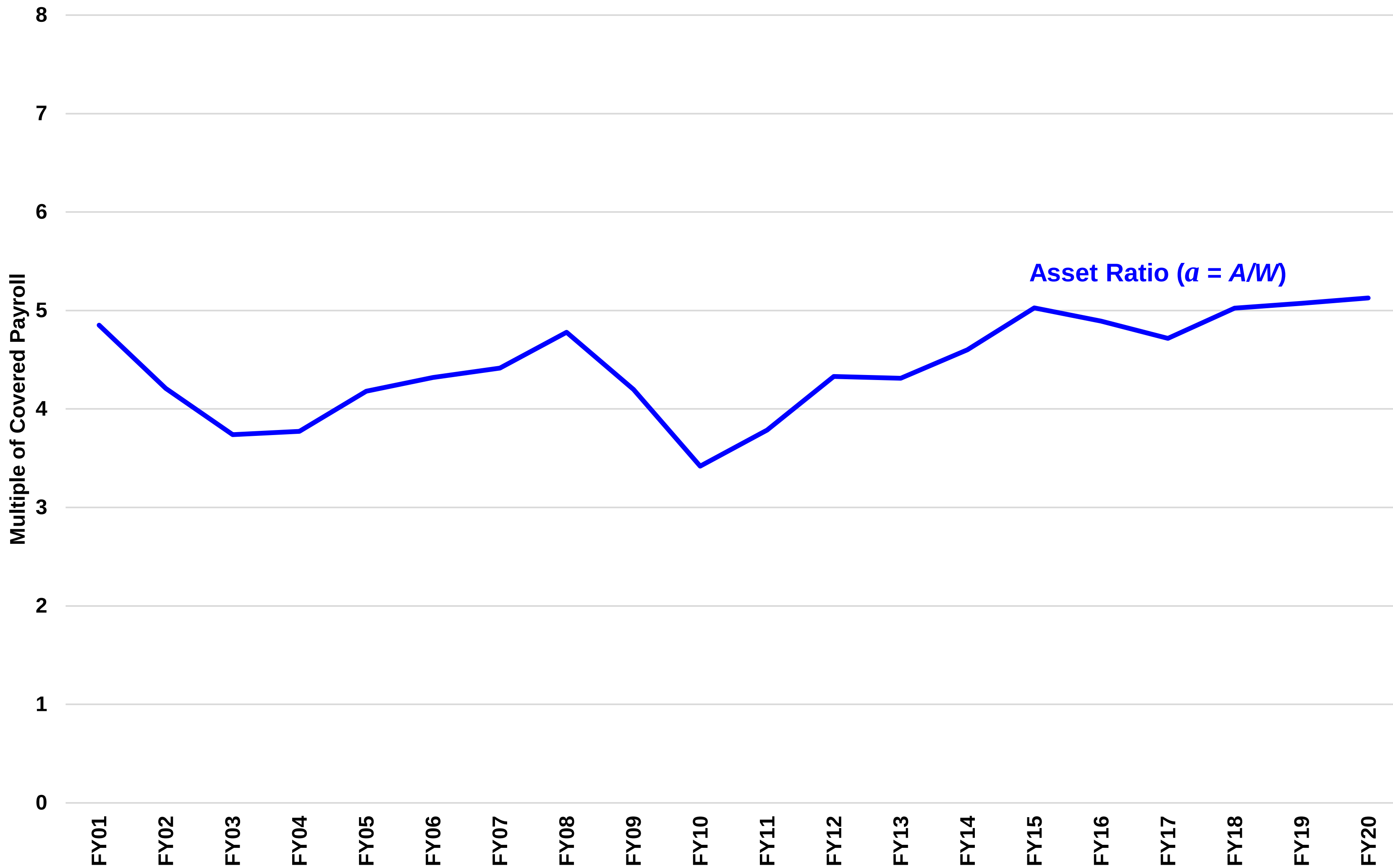
Figure 1. Normal Cost, Contribution and Benefit Rates, FY01 – FY20
Public Plans Data: 119 state & 91 local plans



Source: Center for Retirement Research at Boston College
MissionSquare Research Institute, and National Association of State Retirement Administrators

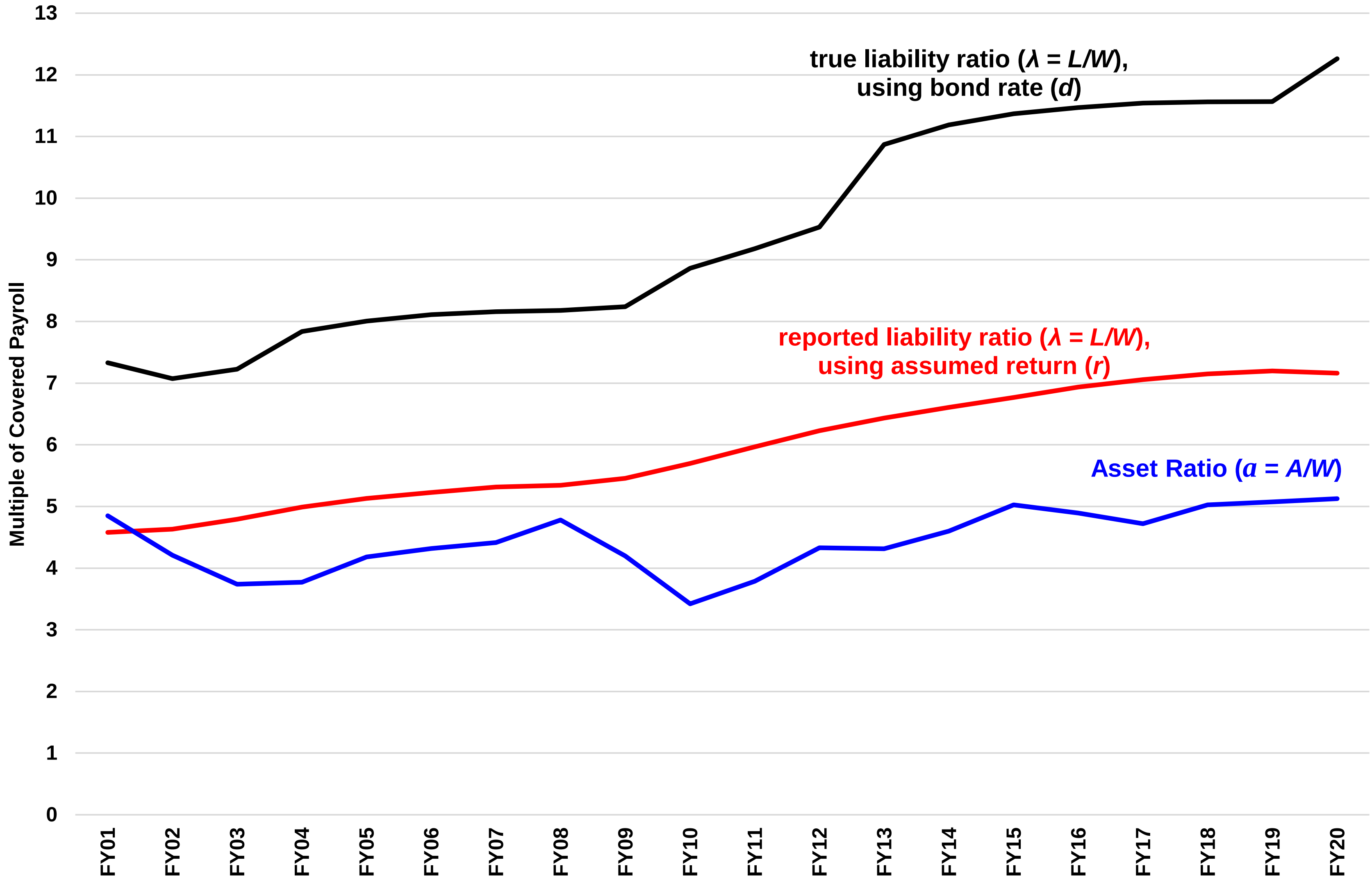
Figure 2. Assets/Payroll, FY01 – FY20

Public Plans Data: 119 state & 91 local plans



Source: Center for Retirement Research at Boston College
MissionSquare Research Institute, and National Association of State Retirement Administrators

Figure 3. Assets & Liabilities, True & Reported, FY01 − FY20



Sources: Center for Retirement Research at Boston College, Federal Reserve Board of Governors & authors' calculations
Both series use PPD payroll

Figure 4: Asymptotic Behavior of Asset Accumulation and Contribution Rate

r = 7%, *g* = 3%

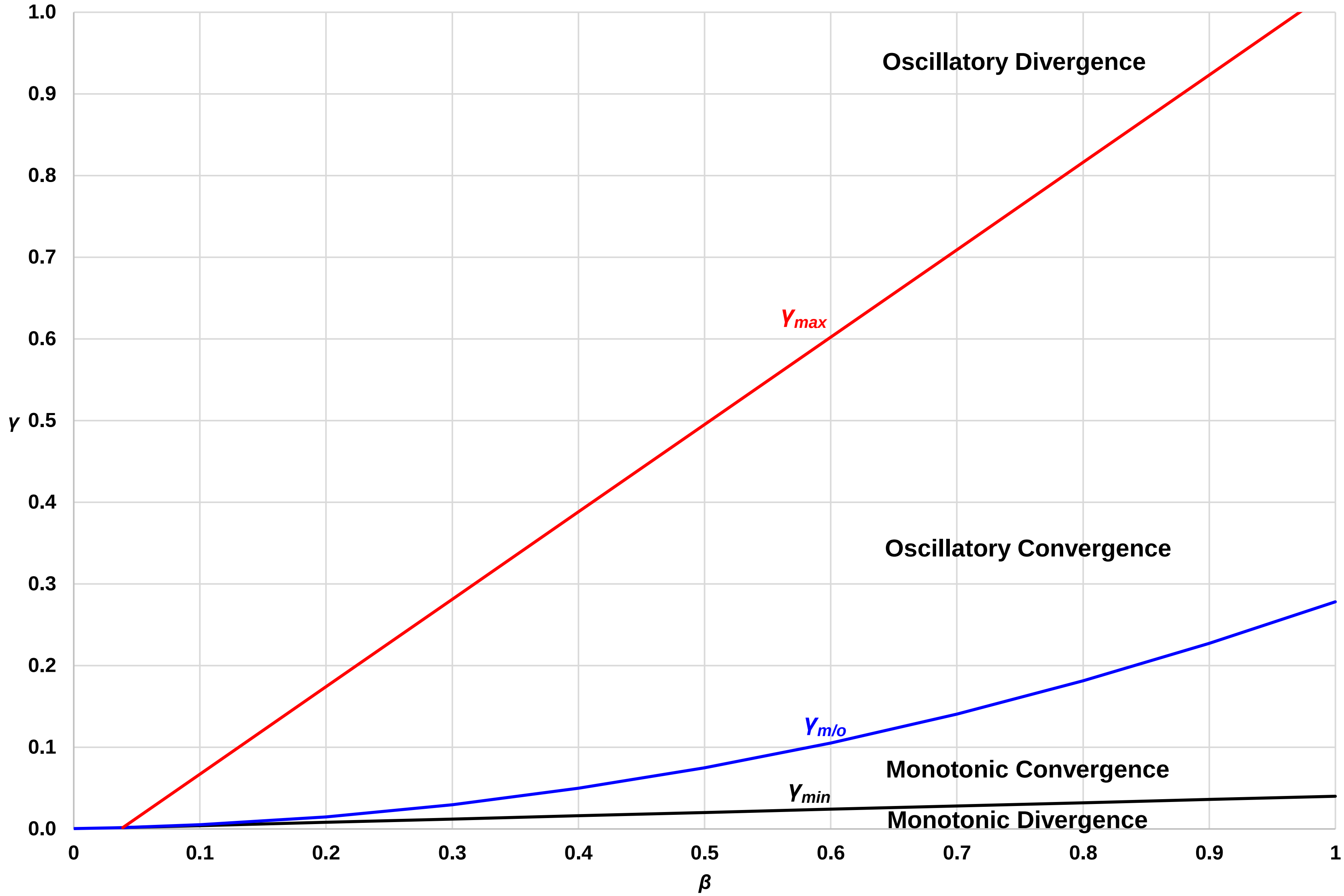


Figure 5. Simulation of Trajectory to "Full" Actuarial Funding

R = 1.07, G = 1.03, a^* = 7.0, c^* = 0.10, β = 0.5, $\gamma = \gamma_{m/o}$ = 0.075

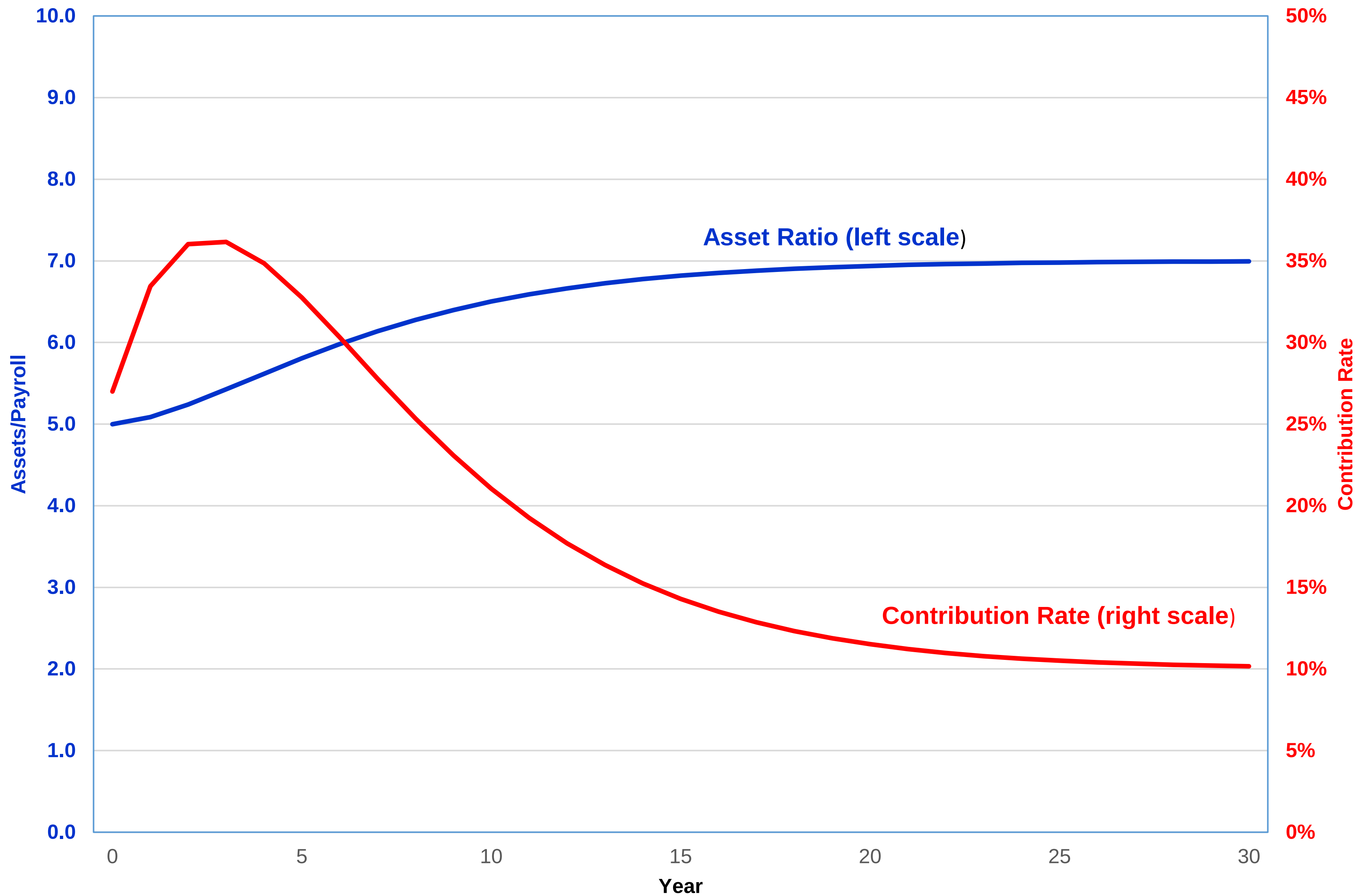


Figure 6. Stochastic Simulation of Trajectory to "Full" Actuarial Funding

R ~ lognormal(μ = 1.07, σ = 0.15), G = 1.03, *a** = 7.0, *c** = 0.10, *β* = 0.5, *γ* = $\gamma_{m/o}$ = 0.075

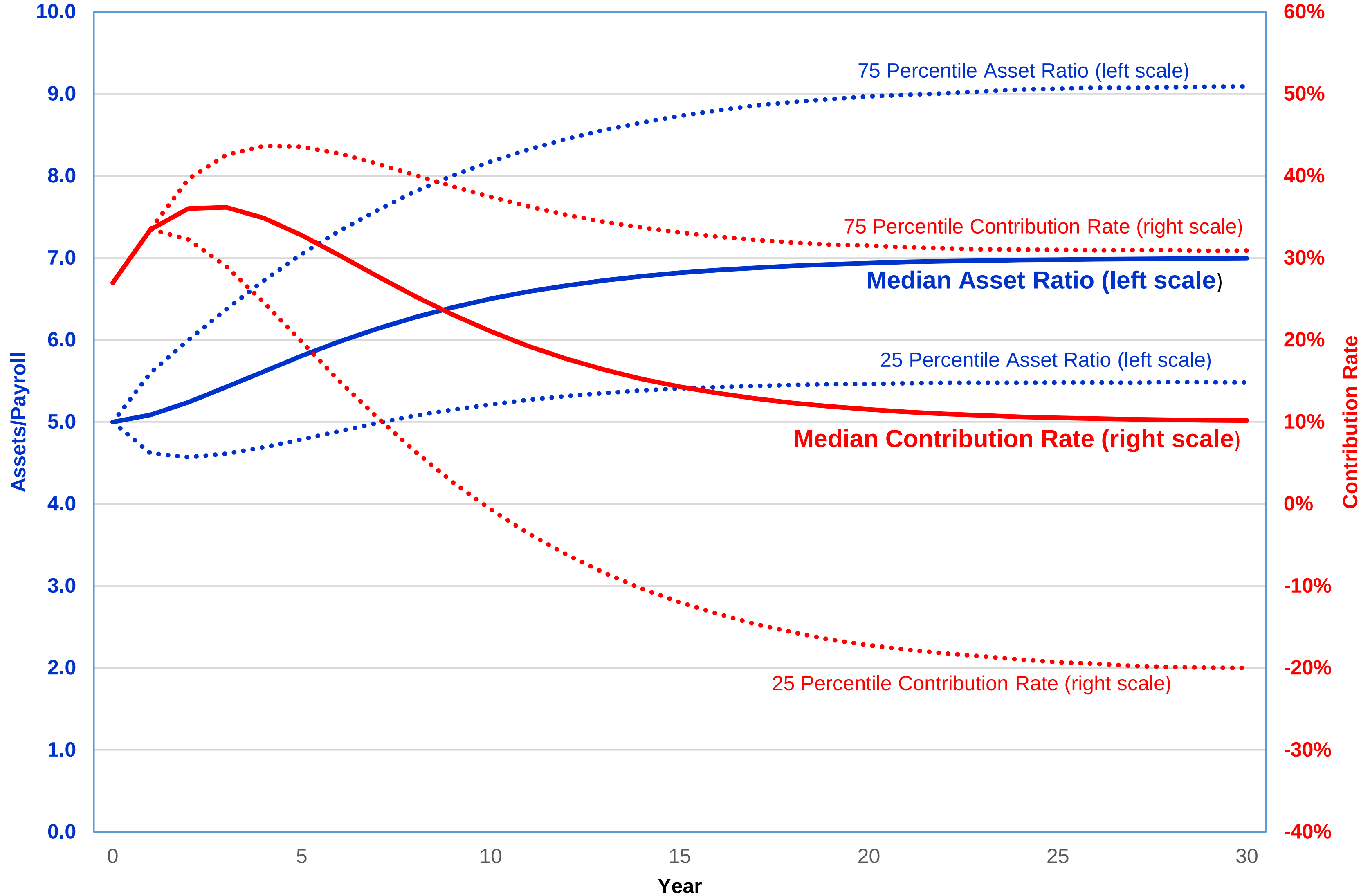


Figure 7. Stochastic Simulation of Slower Trajectory to "Full" Actuarial Funding

R ~ lognormal(μ = 1.07, σ = 0.15), $G = 1.03$, $a^* = 7.0$, $c^* = 0.10$, $\beta = 0.5$, $\boldsymbol{\gamma} = \mathbf{0.0375}$

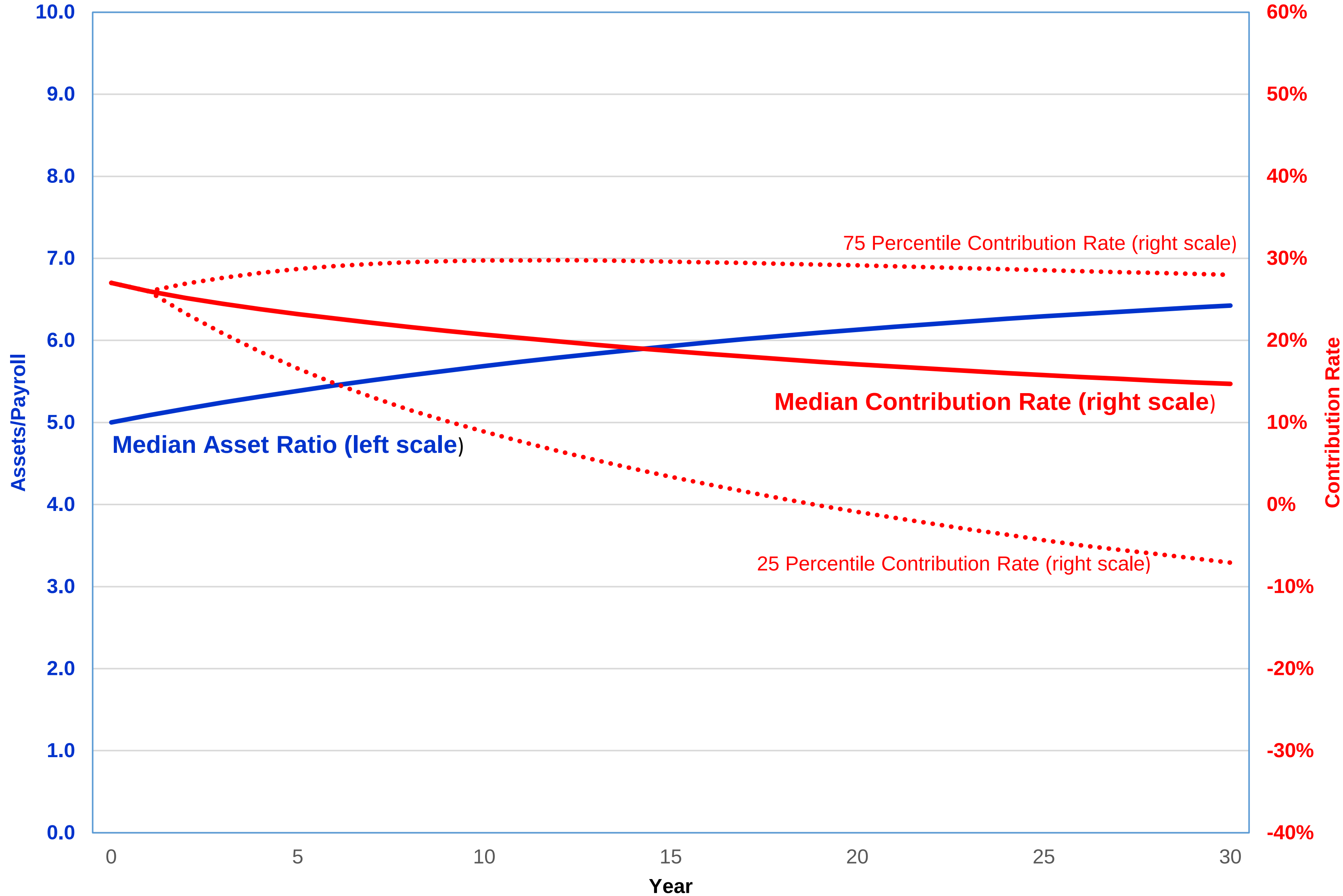


Figure 8A. Median Contribution Rate, Varying Risk and Return

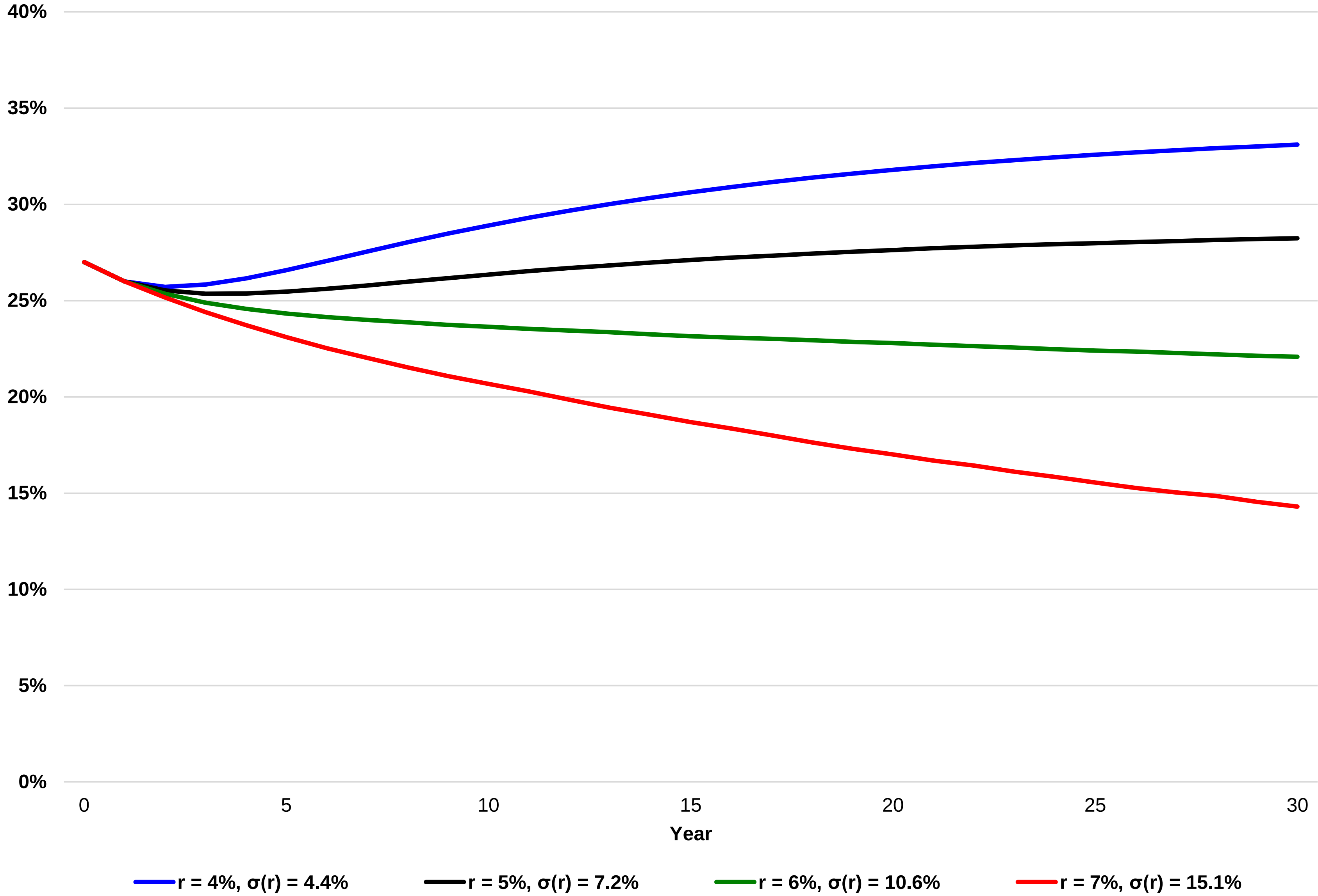


Figure 8B. Standard Deviation of Contribution Rate, Varying Risk and Return

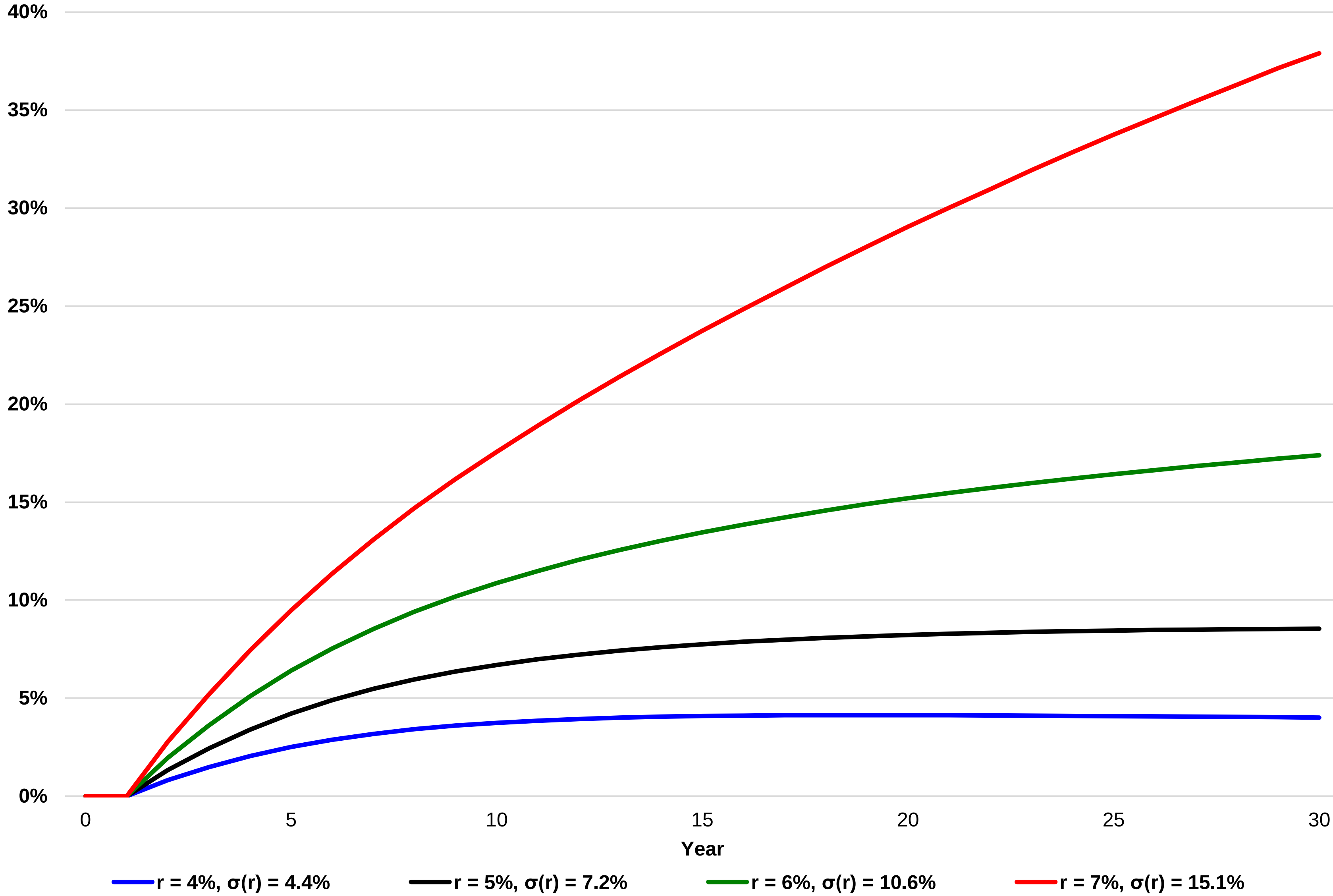


Figure 9: Convexity of Contribution Risk and Annual Return