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EdWorkingPaper No. 23-797

Do Women Still Earn Less than Men after College Graduation: Evidence from the Baccalaureate and Beyond Longitudinal Study 1993 Cohort

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Even though women have continuously caught up with men in education attainment and labor market participation since the 1970s, the wage gap between men and women still universally exists today. Do female college graduates still earn less than their male counterparts if men's and women's "profiles" of observed productivity-related characteristics are statistically adjusted to be equivalent? To answer this research question and better understand the current gender wage gap, I introduce a novel propensity score stratification method for gender wage gap decomposition. This new method overcomes certain limitations of the traditional Blinder-Oaxaca decomposition method, and provides an example of validly applying propensity score-based methods (mostly used in causal settings) to gender wage gap decomposition, a non-causal setting. Making use of this new method, I analyze a nationally representative sample from the Baccalaureate and Beyond Longitudinal Study, which represents the 1993 Cohort of U.S. college graduates. Through propensity score stratification, the observed productivity-related characteristics between men and women in the sample are statistically adjusted to be equivalent within each stratum of propensity score. After "equalizing" these characteristics, evidence shows the women-to-men wage ratio among this college educated population is still 87.4% at the tenth year after they graduated from college. This remaining gender gap cannot be explained by the observed gender differences in productivity-related characteristics, and is the evidence of a discriminatory wage gap possibly existing in the labor market. Additionally, the unexplained gender wage gap universally exists regardless whether these "profiles" of qualifications and labor market experience are stereotypically female or male. Even acknowledging that this research cannot account for all the gender differences in productivity due to data limitation, the results of this research will add to the empirical evidence of measuring the discriminatory wage gap that possibly exists in the labor market.

VERSION: June 2023

Suggested citation: Liang, Mengyuan. (2023). Do Women Still Earn Less than Men after College Graduation: Evidence from the Baccalaureate and Beyond Longitudinal Study 1993 Cohort. (EdWorkingPaper: 23-797). Retrieved from Annenberg Institute at Brown University: https://doi.org/10.26300/14f0-xq26

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Abstract

Even though women have continuously caught up with men in education attainment and labor market participation since the 1970s, the wage gap between men and women still universally exists today. Do female college graduates still earn less than their male counterparts if men's and women's "profiles" of observed productivity-related characteristics are statistically adjusted to be equivalent? To answer this research question and better understand the current gender wage gap, I introduce a novel propensity score stratification method for gender wage gap decomposition. This new method overcomes certain limitations of the traditional Blinder-Oaxaca decomposition method, and provides an example of validly applying propensity score-based methods (mostly used in causal settings) to gender wage gap decomposition, a non-causal setting.

Making use of this new method, I analyze a nationally representative sample from the Baccalaureate and Beyond Longitudinal Study, which represents the 1993 Cohort of U.S. college graduates. Through propensity score stratification, the observed productivity-related characteristics between men and women in the sample are statistically adjusted to be equivalent within each stratum of propensity score. After "equalizing" these characteristics, evidence shows the women-to-men wage ratio among this college educated population is still 87.4% at the tenth year after they graduated from college. This remaining gender gap cannot be explained by the observed gender differences in productivity-related characteristics, and is the evidence of a discriminatory wage gap possibly existing in the labor market. Additionally, the unexplained gender wage gap universally exists regardless whether these "profiles" of qualifications and labor market experience are stereotypically female or male. Even acknowledging that this research cannot account for all the gender differences in productivity due to data limitation, the results of this research will add to the empirical evidence of measuring the discriminatory wage gap that possibly exists in the labor market.

Key words: gender wage gap, propensity score, Baccalaureate and Beyond Longitudinal Study

1. Introduction

Gender wage gap experienced a rapid decrease during the 1970s to the 1980s, a time when the traditional gender roles are challenged. During that time, the number of college graduated women vastly increased, and a growing number of married women stayed in the labor market. Since then, women have continuously caught up with men in terms of education attainment and labor market participation, both of which are viewed as important determinants of one's productivity and earnings. Recent years have even witnessed a phenomenon called "the reversal of gender gap in higher education", where women outnumbering men in college enrollment and completion. These college educated women enter occupations and industries previously dominated by men, and some even take up leadership positions, which further challenges the traditional gender norms. From all the advancement of gender equality mentioned earlier, one might think that the wage gap between men and women must have become insignificant or has even disappeared. However, disappointingly, gender wage gap still exists. The most recent census data show that the female-to-male wage ratio still stalls at the level of 80% or lower, not experiencing a considerable increase in the past two decades. The gender differences in traditional explanatory factors, such as education attainment and labor market participation, have greatly decreased, whereas the gender wage gap is persistent. It suggests that these traditional explanations for gender wage gap might fail to account for the current dynamics of gender wage gap.

Why do women still earn less than men, even though they have equivalent qualifications, if not better? Above all, it should be acknowledged that although women caught up and even outperformed their male counterparts in college, women and men might have diverse trajectories after they graduate from college and enter the labor market. From seeking jobs and promotion, to forming a family and raising children, social norms of gender can make men and women take on different career pathways, cause difference in uncountably many aspects of women's and men's life, and have great influence on their labor supply and productivity. For example, traditional gender norms can become self-fulfilling stereotypes against women. For a very long time, women are expected to take more responsibilities in housework and caregiving, especially after childbirth, while men are expected to be the major breadwinners within households. Hence, even when women are equally committed to their jobs as their husbands or male colleagues, women may still be perceived as less competent and less committed than their male colleagues at work, meanwhile spending a lot more time in housework than their husbands at home. In the long term, this ill perception can lower women's self-motivation at work and wear down women at home, further costing women in productivity. Besides, women's indispensable role in reproduction can lead to a "motherhood penalty", where childbearing and maternal leave(s) disrupt women's working experience accumulation. Such disruption is claimed to negatively affect their overall productivity.

One might argue that it is because women and men are still quite different in all those productivity-related characteristics, especially in labor market experience, that gender wage gap still exists. This leads to the main research question of this study: if statistically adjusting men's and women's "profiles" of observed productivity-related characteristics to be equivalent, do female college graduates still earn less than their male counterparts? To better address this question, this study attempts to use a novel propensity score stratification method that augments the Blinder-Oaxaca decomposition.

Researchers are satisfied with using the Blinder-Oaxaca (BO) decomposition, the most widely used method of gender wage gap decomposition, particularly among econometricians and labor economists. The basic framework of the BO method is to decompose the wage differential into two parts: one can be explained by the gender difference in observed productivity-related characteristics and the other cannot. However, the original BO method makes a linear assumption: the outcome (wage) and the explanatory variables are linearly correlated, and the impacts of the explanatory variables on wage are linearly additive, which is quite strong. Moreover, the BO method can only measure the mean differences of wages between men and women, which makes it not informative regarding to the gender difference across the distribution of the outcome (e.g. wage). Last but not least, the sizes of the "explained" and "unexplained" components estimated from the BO decomposition depend on the choice of the reference group. It means the sizes of the "explained" component and the "unexplained" component can be different between choosing men as the reference group and choosing women as the reference group, which might cause conceptual confusion.

To tackle the limitations of the BO decomposition, I propose to use a novel propensity score stratification method that applies the "balancing" property of the propensity score to gender wage gap decomposition. Compared with the traditional linear regression under the BO framework, this new decomposition method does not need the linear assumption. Meanwhile, through propensity score stratification, this new method can explore the gender wage gaps across the joint distribution of the observed explanatory variables that empirically predicts the distribution of the outcome, and provides additional information that former methods cannot. Finally, within each stratum of propensity score, this new method equalizes the joint distributions of the observed productivity-related characteristics between men and women, which conceptually corresponds to the "explained" part of the gender wage gap being removed. Hence, this method does not need to choose a certain gender as the reference group and can be interpreted more easily.

In this paper, I make use of this new method and analyze a nationally representative data sample from the Baccalaureate and Beyond Longitudinal Study, which represents the 1993 Cohort of U.S. college graduates. From the decomposition analysis, I find that the adjusted women-to-men wage ratio among this college educated population is still 87.4% at the tenth year after they graduated from college. This remaining gender gap cannot be explained by the observed gender differences in productivity-related characteristics, and is the evidence of a discriminatory wage gap possibly existing in the labor market. Moreover, regardless whether these "profiles" of qualifications and labor market experience are stereotypically female or male, the unexplained gender wage gap universally exists across strata of these "profiles" types specified by the propensity scores. The "unexplained" gender wage gaps are significantly different than zero in majority of the strata. In conclusion, even if men's and women's "profiles" of observed productivity-related characteristics are statistically adjusted to be equivalent, female college graduates still earn less than their male counterparts ten years after college graduation, suggesting that a discriminatory gender wage gap might exist among these college graduates in the labor market.

The remainder of the paper is organized as followed: Section 2 reviews the related literature and lays out the uniqueness of this study. Section 3 introduces the methodology for propensity score-based wage gap decomposition. Section 4 describes the data sample and the specification strategy. Section 5 presents the results of the decomposition analysis. Section 6 concludes and discusses the limitation.

2. Literature Review

2.1 Explanation of the gender wage gap

The scholarship studying the determinants of earnings and productivity was pioneered by two economists, Gary Becker and Jacob Mincer. Becker first introduced the concept of “human capital”, which is the investment in human resources that cannot be separated from the people being invested. Human capital investment can take on different forms, including formal education, on-the-job training, health care spending, and so on. Among them, formal education and on-the-job training are the two most common forms of human capital investment, which makes them the two major determinants of laborers’ productivity and potential earnings (Becker, 1964). Corresponding to Becker’s human capital theory that identifies education and training as the major determinants of earnings, Mincer developed an empirical function that laid out the statistical construct for the distribution of personal earnings, which specifies years of schooling and years of working experience as the parameters of the statistical frequency distribution of personal earnings (Mincer, 1974). This function, later named after Mincer, has been widely used to empirically measure the potential earnings of laborers with different human capital profiles consisting of education and training. After the pioneering work of Becker and Mincer, the differences in education attainment and working experience have been the foremost sources when researchers look for explanations for wage gaps, where gender wage gap is not an exception.

Besides years of schooling and working experience, factors directly measuring labor supply or generally related to productivity are also widely used as explanatory factors of the gender wage gap. For example, researchers point out that women and men have diverse patterns of labor force participation after marriage or childbirth. The two gender groups can also be different in average working hours, tenure, turnover, unionization when at work (Cortes and Pan, 2016; Blau and Kahn, 2017). Both imply that women and men have different intensity of overall labor supply. Researchers also found evidence that women and men make different considerations when choosing occupations and industries (Cortes and Pan, 2018). Along with the historical occupational segregation, women end up accumulating in occupations and industries that are less productive (i.e. having lower returns to labor) (Bayard et al., 2003). The gender difference in these factors reflects the gender difference in labor supply and productivity, which contributes to the gender wage gap.

However, after gender parity was reached in the college graduation rate among men and women in the 1980s, female advantages in higher education attainment continue to grow. Women began to outnumber men among college students and college degree earners (Goldin, 1992; Goldin et al, 2006). Meanwhile, staying in the labor market after marriage and childbirth becomes much more common nowadays than in the 1960s and the 1970s (Blau and Kahn, 2007). The whole society are more tolerant and even welcome women to enter some male-dominant occupations and take up leadership positions. Correspondingly, the gender differences in labor market participation and labor supply vastly decrease. Nonetheless, the gender wage gap still exists and the female-to-male wage ratio stalls at 80% or lower, just as the numbers from ten and twenty years ago (Blau and Kahn, 2017). The gender differences in education attainment and working experience has greatly decreased, whereas bridging gender wage gap has shown very little progress. It suggests that traditional factors such as gender difference in education attainment and working experience might fail to provide satisfactory explanations for the current dynamics of gender wage gap.

Consequently, a newer but growing body of scholarship looks for explanations from non-traditional factors such as gender differences in psychological traits, interaction skills, preferences regarding to the career choices, the diverse impacts of social norms on men and women, and so on (Busser et al., 2014; Gneezy et al., 2003; Hilmer et al., 2012). The gender difference in these non-traditional factors are subtler and often hard to measure, but they are claimed to cause a hidden difference in productivity between men and women. It should be noted that in the traditional gender wage gap decomposition, after accounting for the gender differences in the observable factors that affect productivity and labor supply, the remaining unexplained wage differential is attributed to gender discrimination. If more previously unobserved non-traditional factors can be additionally included, part of the previously unexplained component of the gender wage gap will be accounted for and will provide a more accurate decomposition result. Although consensus has not been reached on how to validly measure these non-traditional factors, some researchers found empirical evidence on the impacts of these factors on productivity and labor supply from small-scale experiments. A small to moderate contribution of these factors to the gender wage gap has been found in these studies (Blau and Kahn, 2017). Meanwhile, some large-scale surveys begin to include questions about the decision-making process and self-reported psychological traits. However, this field of research is still far from being mature due to the limited availability of data in surveys and the challenges of quantitatively measuring these factors.

Although not including information of the non-traditional factors, data from the Baccalaureate and Beyond Longitudinal Study includes very detailed measurement of the subjects' demographics, education experiences, career trajectory, and family formation, which other datasets rarely have. Making use of this dataset, this study will be able to account for fine-grained differences between men and women in the ten-year trajectory from studying in colleges to working in the labor market, which previous studies could not. However, as the survey did not include questions about the non-traditional factors, this research will only account for the observed productivity-related characteristics (mostly traditional factors) available in the data and provide a conservative estimation of the discriminatory gender wage gap for the certain survey population.

2.2. The Blinder-Oaxaca (BO) framework

To most common way to understand and measure gender wage gaps is to use the Blinder-Oaxaca decomposition, which was originally proposed by Blinder (1973) and Oaxaca (1973). The main idea is to decompose the wage differential between two mutually exclusive demographic groups (e.g. men and women, white and non-white) into the "explained" component and the "unexplained" component. Detailed derivation of the BO method is presented in the appendix section. Limitations of the original BO decomposition method include the following. Firstly, it only accounts for the wage differential in the level of group mean average. Consequently, it is not informative regarding the gender wage gap across the distributions of wage, both within gender group and between gender groups. Secondly, the BO method uses a log-linear wage function, which assumes a linear relationship between the explanatory variables (productivity-related characteristics) and the outcome (logarithm of wage), and the impacts of the explanatory variables are linearly additive. This linearity assumption is too strong and can oversimplify the relationship. Finally, the size of the components estimated from the BO decomposition is largely based on the reference group, which means the size of the "explained" component and the "unexplained" component may not be consistent when choosing different reference groups, which will cause conceptual confusion.

To overcome the first and the second limitations mentioned above, previous researchers extended the BO decomposition to allow different forms of regression modeling for the outcome. For example, using a quantile regression model for the outcome (wage), researchers can examine wage gaps at different percentiles of the wage distribution (Juhn et al, 1993). Using a general conditional expectation function for the outcome model, researchers allow non-linear relationship between outcome and its explanatory variables (Barsky et al, 2002). Moreover, the outcome variables are not restricted to be wage/earnings, as gender gaps may exist in the probability of getting promoted or getting the management positions. To measure these types of gender gaps, the outcome variable may be binary, e.g. whether becoming a manager or getting promoted. To decompose the gender gap in binary outcomes, probit or logistic regression can be used for the modeling of the outcome (Yamaguchi, 2016). If only interested in the dynamics of the outcome(wage), using weighted kernel density estimation method by incorporating the explanatory variables to the weights, the outcome function does not even need to be parametrically specified (DiNardo, Fortin, and Lemieux, 1996).

Inspired by and built on the previous methods under the BO framework, my new method also tackles the limitations of the original BO method mentioned above, and attempts to validly use propensity score based methods in gender wage gap decomposition, an inherently non-causal setting. Firstly, this new method does not have assumptions on the functional form of the outcome. Secondly, this new method does not need to choose the reference group. Instead, through propensity score stratification, the new method "equalizes" the joint distributions of the observed productivity-related characteristics between men and women within strata of propensity scores, which can be easily conceptualized. Lastly, stratification also makes the decomposition informative regarding to gender wage gaps across the joint distribution of the explanatory variables that empirically predicts the distribution of the outcome. In other words, this new method allows researchers to explore gender wage gaps within and across strata of the propensity scores. Details about the new method is described in the following section.

3. Methodology

To better address the research question about the current gender gap and better understand it, I propose a new decomposition strategy that uses propensity score stratification to "equalize" the joint distributions of multiple observed explanatory variables between men and women. Other propensity score based methods, such as propensity score based matching and weighting would be additional alternatives. I choose to use propensity score stratification because it allows me to explore the "unexplained" gender wage gap across strata of "profiles" of observed productivity-related characteristics specified by propensity scores. The rationale of this method is to utilize the fundamental balancing property of the propensity score that can equalize the joint distribution of covariates.

Propensity score is a concept first proposed by Rosenbaum and Rubin (1983), where the goal was to remove bias of the treatment effect in observational studies due to all observed covariates. Such bias would arise if there exist pre-treatment differences of the covariates between the treatment group and the control group. Since firstly published, propensity score-based methods have been broadly used to estimate the causal effects of treatments in non-experimental designs, such as observational studies, social "experiments" or policy analysis.

Essentially, the propensity score is a balancing score. It is a function of the observed covariates X and has the property of "balancing" the joint distribution of these covariates between the treatment and control groups. A function is a balancing score of X if the conditional distributions of observed variables X given the balancing score are the same between the treatment group (Z=1) and the control group (Z=0). In other words, conditioning on the balancing score, the distributions of the observed covariates X are equivalent for the treatment and control groups. Given the propensity score $e(X)$, the distribution of the observed covariates X is independent of the treatment assignment Z:

$$X \perp Z \mid e(X)$$

$$e(X) = \Pr(Z = 1|X)$$

Decomposing the gender wage gap is inherently different from evaluating the effect of a treatment. Gender is not a "treatment" as in the causal setting because it cannot be manipulated, and gender wage gap decomposition is a descriptive analysis. However, utilizing the "balancing" property of the propensity score does not need causal claim, since it only assumes the equivalence of the joint distributions of X between two groups. Under the BO framework, the "explained" component of the gender wage gap are the part that equal productivity and labor supply will lead to equal pay, where the productivity-related characteristics predicts the "explained" wage. In other words, if productivity-related characteristics are equal, the "explained" part of the wage are equal between the two gender groups, so the condition of $e(X)$ being of "balancing" property is satisfied. Also, under the BO framework, the remaining part of the gender wage gap is the "unexplained" component. Correspondingly, conditioning on the propensity score, the "explained" part of the gender wage gap has been equalized or removed. If there is any remaining gender wage gap, this part may be attributed to unobserved factors or gender discrimination. The above rationale enables the application of propensity score-based methods to the gender-based decomposition analysis without causal claims.

To better understand how to apply the propensity score based methods usually used in causal settings, the relationships between gender, explanatory variables and wages can be interpreted in a way analogous to treatment, covariates, and outcomes in the causal setting. In this analogy, the indicator for gender corresponds to the indicator for treatment assignment, where being a man is analogous to being in the treatment group, while being a woman is analogous to being in the control group. The "balancing" property of the propensity score here is:

$$X \perp N \mid e(X)$$

$$e(X) = \Pr\,(N = f|X)$$

where X denotes the productivity-related characteristics and N denotes the gender indicator, the joint distributions of X are equivalent between two gender groups given the propensity score $e(X)$, which is satisfied as previously stated.

As the decomposition analysis gender wage gap is non-causal, $e(X)$ is hard to conceptualize. I propose to interpret it as the probability of being a woman given certain observed productivity-related characteristics. This interpretation is easy to understand conceptually. Suppose the employers in the market can observe all productivity-related characteristics X of employees and has the information on the proportion of women given the observed "profile" of these characteristics, which is $e(X)$. However, the employers cannot observe the gender of a certain employee and can only guess based on the information. A labor market without gender discrimination will ignore the information on the conditional proportion of gender and will determine the wage levels purely according to the observed “profile” of productivity-related characteristics. However, a labor market with some degree of gender discrimination will consider the conditional proportion of gender and will assign different wages based on the conditional probability of this “profile” being a certain gender. As stated before, controlling the conditional probability of gender, i.e. the propensity score and ensuring no significant within-stratum difference of the joint distributions of observed variables, the “explained” part of the wage gap by the observed variables can be removed. Corresponding to the BO framework, the remaining difference is the “unexplained” part of the gender wage gap. Henceforth, any remaining gender wage gap after controlling for the propensity score can be viewed as the evidence of gender discrimination possibly existing in the labor market.

From the above rationale, we can apply propensity score-based methods, including propensity score matching, weighting and stratification to the gender-based decomposition analysis without causal claims. The analytic results of using propensity score stratification to measure gender wage gap will be presented in the section 5.

This way of combining the propensity score with the BO framework is advantageous. First, the “balancing” property of the propensity score significantly reduces the multi-dimensional complexity of the explanatory variables. Second, the adjusted wage gap through propensity score based methods are easy to understand conceptually and does not need to choose a reference group. Specifically, we can relate the remaining gender wage gap to the “unexplained” part of the gender wage gap. Third, propensity score based methods allow researchers to explore the “unexplained” gender wage gaps across the joint distributions of the observed productivity-related characteristics specified by propensity scores. Last but not least, the propensity score based methods allow flexibility of the functional form for the outcome model.

While applying the propensity score based methods to the gender wage decomposition is advantageous, it also has underlying limitations. The propensity score methods are only effective within the common support of the propensity scores. In other words, it excludes those people who are beyond the range of propensity score levels shared by both gender groups from the analysis. Moreover, even though the functional form for the outcome model can be non-parametric, the propensity score models still need to be correctly specified. Hence, if the functional form of the propensity score is incorrect, the validity of this method will suffer. Besides, propensity score based decomposition can also be biased if confounders and important unobserved explanatory variables exist. In other words, although it is not attributable to the observed productivity-related characteristics, we cannot claim the “unexplained” part of the gender wage gap is completely attributable to gender discrimination, because the non-traditional factors such as preferences, personal choices, and psychological traits are still not included in the analysis. If the aggregate impact of these unobserved non-traditional factors on productivity are higher among women, then the women’s productivity is underestimated by the current model, which means the gender discrimination is underestimated. On the contrary, if the aggregate impact of these unobserved non-traditional factors on productivity are higher among men, then the women’s productivity is overestimated by the current model, which means the gender discrimination is overestimated. As these non-traditional factors are unobserved and their impacts on women’s and men’s productivity are unknown, the adjusted wage gap is interpreted as a conservative estimation of the gender discrimination possibly existing in the labor market.

4. Sample

4.1 Dataset

I analyze the Baccalaureate and Beyond Longitudinal Study (B&B) that followed 11,192 college graduates and gathered information on their demographics, education experiences, employment trajectory, family formation, and other aspects of their lives through multiple waves of data collection. The sample for this longitudinal study was drawn from the National Postsecondary Student Aid Study (NPSAS) by the National Center for Education Statistics in the Institute of Education Sciences.

The B&B data is informative in many ways. First of all, the B&B study includes a nationally representative sample of the college seniors in the United States in 1993 who entered the labor market during the 1990s, the period that the labor market showed the paradoxical trends. Secondly, the B&B study contains longitudinal data for baseline and the following three waves, which are collected in the senior year of the college students, the first, the fourth and the tenth year after graduation. The three waves of data collection on their labor market experiences during the first ten years correspond to three different stages of one's early career. Last but not least, the B&B data has rich information on both the college graduates' educational experiences during college and their employment history, work-related experience, and family formation during the first ten years in the labor market. We can use the data to examine the evolution of the gender wage differential over the 10-year period and to investigate the contribution of the educational experiences to the gender wage differential at the early stage of the early career.

4.2 Specification strategy

The outcome variable is the logarithm of the annual salary of the subjects in 2003, which is the tenth year after their college graduation. In the B&B data, the subjects' annual salary in 2003 have been adjusted to be on the same scale (annually) from the original records of their wage rates under different basis (hourly, weekly or monthly). This helps to reduce the large noise in the subjects' actual annual earnings caused by their different labor market participation and corresponding wage basis.

The explanatory variables for gender wage gap used in this study can be categorized into three groups, based on the time that the gender difference of these variables came into existence:

First, the "labor market experience" difference that existed after college graduation, including the on-the-job training received in total, the highest post-graduate degrees earned, the career path, the unemployment history, and years in the industry. The career path is measured using the survey question, "whether the current job has career potential", consecutively asked in the first two waves. The unemployment history is measured by the times of unemployment experienced in the ten years after graduation. The "current" productivity-related characteristics in 2003 include the "current" labor market participation, the occupation, industry and sector of the "current" employment, the "current" marriage status and whether had paid leave for children.

Second, the "college experience" difference existed during college. These include the students' major GPA, whether in Science, Technology, Engineering, Mathematics (STEM) majors, the selectivity of the institution, and the number of the internships or apprenticeship experience. Moreover, certain cutoffs are usually used to evaluate the student's school performances based on their GPA in actual recruiting practices. Accordingly, I create a categorical variable to reflect the reality of the GPA cutoffs. "Low" GPA is defined as lower than 3.0, "high" GPA is defined as higher than 3.5, and the "Medium" GPA for is 3.0 to 3.5.

Third, the “demographics/background” difference that existed before college, including variables such as race/ethnicity, socio-economic status indicators, SAT/ACT quartiles. The socio-economics status indicators are measured by two variables, the highest education level of the parents, and the combined category of independence status and income quartiles. Since the data sample is representative of all college graduates in 1993, the women and men in this sample can be incomparable due to these background differences as two sub-groups. For instance, in the data sample, female college graduates are more likely to be non-white. Including the variables such as race/ethnicity can reduce potential selection bias that make the comparison less relevant to gender difference.

The descriptive statistics of the variables used in the analysis is presented in Table 1.

Table 1. Descriptive Statistics of the Variables Included in the Analysis

VARIABLES	Men	Women
Gender (proportion of the whole sample)	0.43	0.57
Outcome variables:		
Annual Salary in 2003	54,923.69	38,663.06
Explanatory variables:		
1. “labor market experience"		
Labor force participation 2003 (%):		
Full-time, one job	63.99	48.21
Part-time, one job	2.82	9.92
Multiple jobs	7.94	7.64
Unemployed	2.74	2.92
Out of the labor force	1.62	12.32
Number times unemployed since 1997	0.19	0.17
Number of unemployment spells	0.53	0.53
Years pursuing career in industry	8.79	9.32
Occupations (%)		
Management	14.71	11.17
STEM occupations	17.38	6.08
Teacher	7.88	20.47
Medical professionals	5.39	10.44

Law professionals	3.10	2.27
Business	11.64	10.00
Job in desired industry/occupation (%):		
Job not part of a career	9.23	10.88
Job part of a career	84.69	69.57
Degree of career potential at 1997 April job (%):		
Definite career potential	49.91	43.44
Possible career potential	22.75	25.54
Not much career potential	13.42	17.30
Degree of career potential at 1994 April job (%):		
Definite career potential	35.01	31.15
Possible career potential	25.27	27.91
Not much career potential	22.29	25.26
Highest degree attained by 2003 (%):		
Bachelor's degree	55.80	56.58
Post-baccalaureate certificate	0.75	0.98
Master's degree	15.27	18.76
Post-master's certificate	0.27	0.44
First-professional degree	4.42	2.94
Doctoral degree	2.61	1.31
Classes: earned credits, while at work (%):		
Did not earn college credits	56.03	52.91
Earned college credits	40.27	41.57
Total hours spent in training last year, 1997	64.25	48.85
Hours spent in training last year, 1994	27.89	21.89
Marital status, 2003 (%):		
Single, never married	16.59	14.41
Married	54.62	55.79
Cohabiting/living with a partner	3.55	3.65
Separated	0.73	1.12
Divorced	3.57	5.47
Widowed	0.06	0.58
Leave for children since 1997: any taken (%):		
Took leave for children	38.87	24.92
Did not take leave for children	11.84	28.04
Not applicable - no children	27.63	24.66
2. "College experience"		
Number of internship, apprenticeship in 1992-93	0.30	0.40
Major GPA (%)	3.26	3.37
High GPA	44.9	56.1
Medium GPA	40.5	33.9
Low GPA	14.6	10
Flag indicating if honors awarded (%):		
Honors Mentioned in Transcript	15.82	22.25

Institution selectivity (%)		
Doctoral Universities	48.97	40.28
Master's Colleges and Universities	31.76	39.41
Baccalaureate Colleges	14.79	14.91
Baccalaureate/Associate's Colleges	0.12	0.16
Associate's Colleges	0.10	0.95
Special Focus Institutions	3.19	3.74
STEM majors (%)	31.80	12.23
3. “Demographics/background”		
Parent’s highest education attained (%):		
Less than high school graduation	2.82	4.30
High school graduation or equivalent	18.33	19.00
Vocational/Trade/Business school	6.41	8.38
Associate’s degree or some college	11.7	12.91
Bachelor’s degree (4-5 year)	23.15	22.32
Master’s degree or equivalent	22.04	18.91
Doctorate (Ph.D., Ed.D.)	4.98	3.68
Family income quartiles by dependency (%):		
Dependent, 1st quartile	13.81	17.33
Dependent, 2nd quartile	14.75	15.05
Dependent, 3rd quartile	14.48	12.94
Dependent, 4th quartile	14.52	13.14
Independent, 1st quartile	12.82	9.90
Independent, 2nd quartile	12.72	9.43
Independent, 3rd quartile	9.09	10.76
Independent, 4th quartile	6.04	9.67
Race/ethnicity (%):		
American Indian/Alaska Native	0.39	0.71
Asian or Pacific Islander	4.67	3.17
Black, non-Hispanic	4.17	6.29
Hispanic	4.25	5.02
White, non-Hispanic	76.33	73.72
Merged SAT and ACT score quartile (%):		
Bottom quartile	16.10	22.87
Second quartile	19.44	22.26
Third quartile	23.40	17.56
Top quartile	24.27	14.99

Note: Missing categories (blank, legally skip, or non-response) are not listed. For this reason, some proportional variables do not add up to 100%.

5. Analytic results

5.1. Descriptive analysis

Table 2 shows the descriptive statistics of the gender composition and annual salary for the full sample and the analytic sample which excludes missing in the outcome (the natural logarithm of the 2003 annual salary).

From the descriptive statistics for the full sample, we can see that among the 11,152 students graduating from college in 1993, 56.8% of them are women. However, ten years after graduation, these women on average earn $38,638 per year, which is only 70.39% of the annual salary of their male counterparts.

The analytic sample was constructed by excluding the subjects who did not report positive annual salary in 2003 or beyond the common support of the predicted propensity score. 206 subjects have a predicted propensity score beyond the common support of women and men sub-samples. Among the subjects within the common support, 8,543 (78.5%) subjects have reported positive annual salary in 2003 and they comprises the analytic sample.

A comparison between the B&B full sample and the analytic sample suggested some small differences in gender composition and women's average annual salary. After excluding the people beyond the common support and not reporting positive annual salary, the proportion of women in the analytic sample is lower than their proportion in the full sample by 0.1%. We can postulate that some of these college educated people in the B&B survey did not participate in the labor force actively ten years after graduation and a larger proportion of them are women. Possibly due to this reason, the average salary of women in the analytic sample is higher than that in the full sample, which drives up the unadjusted wage ratio to 70.78%.

Table 2. Comparison between B&B Full Sample and Analytic Sample

VARIABLES	Full Sample	Analytic Sample
Observation	11,152	8,543
Proportion of Women	0.568	0.567
Outcome Variable (natural log):		
Average Salary of Women	38638.33(10.562)	38870.85(10.568)
Average Salary of Men	54940.17(10.914)	54940.17(10.914)
Wage Ratio (Unadjusted)	70.39%	70.78%

5.2. Decomposition analysis

To answer the research question on how much does female college graduates earn less than their male counterparts, I use the propensity score stratification method. The regression model used to predict propensity score, $e(X) \equiv Pr(female = 1|X)$, is:

$$\ln\left(\frac{Pr(female = 1|X)}{1-Pr(female = 1|X)}\right) = a_0 + a_1 X,$$

where X is the vector of all the covariates, including all the productivity-related characteristics listed in Table 1.

The stratification is applied through the STATA command "pstrata" (Linden, 2016). The stratification generates 21 strata. The within-stratum balance in the predicted propensity score is reached for women and men (presented in Table 3). From stratum 1 to stratum 21, the average propensity score is increasing, which implies that the "profile" of the observed productivity-related characteristics is more likely to be men's in stratum 1, and is more likely to be women's. The within-stratum balance is also reached in the explanatory variables, with none of the explanatory variables has statistically significant difference by gender within stratum.

To ensure the conditional probability of being female given the observed "profile" of each stratum is equivalent to that of the entire analytic sample, each of the regression adopts a strategy of marginal mean weighting through stratification (MMWS) (Hong, 2010). The strategy is applied through the STATA command "mmws" (Linden, 2014) through applying the following weight to each subjects in the sample:

$$MMWS = \frac{Pr(female = 1|X)}{Pr(female = 1|X, Stratum = s)}$$

where s=1,2,…,21 and represents the 21 strata.

Table 3. Within-Stratum Balance in the Predicted Propensity Score for Women and Men

	Women			Men		
Stratum	N	Mean	SD	N	Mean	SD
1	38	0.068	0.004	484	0.065	0.001
2	71	0.132	0.005	450	0.137	0.001
3	88	0.201	0.005	433	0.201	0.001
4	140	0.259	0.005	381	0.263	0.001
5	170	0.318	0.005	352	0.321	0.001
6	192	0.373	0.004	329	0.376	0.001
7	203	0.420	0.004	318	0.422	0.001
8	239	0.463	0.004	282	0.466	0.001
9	280	0.506	0.004	242	0.508	0.001
10	301	0.546	0.004	220	0.548	0.001
11	307	0.583	0.004	214	0.587	0.001
12	317	0.619	0.004	204	0.623	0.001
13	3337	0.656	0.004	185	0.658	0.001
14	373	0.694	0.004	148	0.698	0.001
15	402	0.736	0.004	119	0.739	0.001
16	413	0.778	0.004	108	0.782	0.002
17	430	0.822	0.005	92	0.825	0.002
18	458	0.868	0.005	63	0.869	0.002
19	466	0.907	0.005	55	0.910	0.002
20	482	0.941	0.005	39	0.944	0.002
21	508	0.971	0.005	13	0.974	0.003

The joint distribution of the observed productivity-related characteristics are equalized within stratum through propensity score stratification, or more specifically, through applying the above MMWS weights when running the analytic regression models. The models I run for the whole sample population and sub-sample populations are in the following form:

$$\ln(W_i) = \beta_0 + \beta_1 female_i + u_i$$

where β_1 is the coefficient of interest, which represents the gender difference of annual salary in the percentage of the men's annual salary; besides, subscript *i* (i=1,2,3,…, n) represents the n subjects in the analytic sample; *W* represents the annual salary in 2003; *G* represents the major GPA; *female* is the indicator variable for being woman; β_0 is the average annual salary for men in natural logarithm; u_i is the disturbance term, where $u_i \sim N(0, \sigma^2)$.

The analytic results show that after adjusting all the observed productivity-related characteristics through propensity score stratification, the remaining gender wage gap of the whole analytic sample is 12.6% of the men's annual salary. Alternatively stated, it means that after adjusting for the all the observed productivity-related characteristics, the women-to-men wage ratio is 87.4% and is statistically significant different than 100%. Compared with the unadjusted women-to-men wage ratio (70.78%), the adjusted wage ratio is a lot larger. Apparently, the observed productivity-related characteristics cannot completely explain the gender wage gap. However, as previously mentioned, gender difference in psychological traits and personal choices based on job preferences are unobserved in the data and can have important explanatory power of the gender wage gap. With the potential existence of these important unobserved explanatory variables and the aggregate effects of these unobserved factors on women and men are unknown, it is too aggressive to claim that adjusted gender wage gap is the exact measurement of the existing gender discrimination. Regardless, it can be viewed as a conservative estimation of the discriminatory gender wage gap before more explanatory variables are included and accounted for in further analysis.

The estimated measurement of gender gap in certain sub-sample populations are also reported in Table 4. The results show that, among students of minority racial/ethical groups, the women-to-men wage ratio is 93.3% and not significantly different than 100%. Alternatively stated, the wage levels of women college students of minority racial/ethical groups might not be different from that of their male counterparts.

Then I focus on the disadvantageous factors from family background. Among first-generation students, the women-to-men wage ratio is 87.5% and significantly different than 100%. In other words, the estimated gender wage gap among first-generation students is very much alike that of the whole sample. If focusing on whether coming from low-income families as the disadvantageous factor, the gender wage gap is larger compared with the whole sample.

When looking at people in different GPA groups, each one of the sub-populations has a women-to-men wage ratio that is significantly different than 100%. The women-to-men wage ratios are 89.1%, 86%, and 84.6% respectively for people who got high GPA (>3.5), medium GPA (3.0 to 3.5) and low GPA (<3.0). It should be noted that observing from the constant terms, the high GPA group of people did not necessarily earn the most. Nonetheless, when compared with the other two GPA groups, the high GPA group has the largest women-to-men wage ratio, i.e. the smallest gender wage gap. For subjects who were in STEM majors, the women-to-men wage ratio is 91.1% and not significantly different than 100%. Meanwhile, judging from the constant term, their average annual salary is much higher than the whole sample. This means majoring in STEM majors might greatly help narrow the gender wage gap.

Table 4. The Analytic Results for the whole sample and sub-sample populations

VARIABLES	Whole Sample	Sub-samples		
		Non-white	First-generation	Low-income
Gender (Female=1)	-0.126***	-0.067	-0.125***	-0.145***
	(0.0227)	(0.0711)	(0.0287)	(0.0264)
Constant	10.78***	10.73***	10.82***	10.75***
	(0.0198)	(0.0645)	(0.0233)	(0.0211)
Observations	8,543	1,190	4,451	4,471
R-squared	0.008	0.002	0.007	0.011

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 4. (Continued)

VARIABLES	Sub-samples			
	High GPA	Medium GPA	Low GPA	STEM majors
Gender (Female=1)	-0.109***	-0.140***	-0.154***	-0.089
	(0.0355)	(0.0347)	(0.0524)	(0.0570)
Constant	10.77***	10.81***	10.74***	10.91***
	(0.0320)	(0.0298)	(0.0393)	(0.0488)
Observations	4,210	3,272	1,061	1,786
R-squared	0.006	0.010	0.013	0.004

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

As previously stated, gender wage decomposition using propensity score stratification allows us to explore the difference existing in the outcome distributions for the whole sample population. In Table 5 and Figure 1, I present the within-stratum gender difference in 2003 annual salary across strata. From the results, we can see that the gender difference universally exists in all strata and majority of them are statistically significant. Looking at the average 2003 annual salary across strata, we can see that from stratum 1 to stratum 21, the within-stratum average annual salary is mostly decreasing, that is, if the "profile" of observed productivity-related characteristics is more likely to be men's, the average annual salary is higher. Henceforth, the gender wage gap exists both within stratum and across stratum.

Table 5. Within-Stratum Difference in 2003 Annual Salary between Women and Men

	Wage Ratio	Average Wage	Women			Men		
Stratum			N	Mean	SD	N	Mean	SD
1	92%	60994	38	56599	0.08	484	61348	0.03
2	85%**	64174	71	55741	0.06	450	65634	0.02
3	76%***	60909	88	48784	0.06	433	63829	0.03
4	89%**	58134	140	53197	0.05	381	60084	0.03
5	84%***	53491	170	47692	0.05	352	56804	0.04
6	88%**	53217	192	49004	0.04	329	55770	0.04
7	85%**	53943	203	48995	0.05	318	57723	0.05
8	84%***	50915	239	46306	0.04	282	55380	0.04
9	92%	50458	280	48400	0.05	242	52746	0.07
10	83%**	45868	301	42457	0.05	220	51415	0.07
11	81%***	45154	307	41177	0.04	214	50881	0.06
12	93%	44258	317	42979	0.05	204	46207	0.05
13	83%**	43057	3337	40351	0.03	185	48463	0.05
14	92%	42064	373	41044	0.03	148	44582	0.07
15	90%*	38479	402	37503	0.03	119	41900	0.06
16	85%**	39967	413	38690	0.03	108	45413	0.06
17	89%	36565	430	35829	0.04	92	40147	0.06
18	117%	35953	458	36645	0.03	63	31300	0.17
19	110%	33125	466	33504	0.04	55	30325	0.19
20	88%	32595	482	32283	0.03	39	36829	0.10
21	68%***	30772	508	30493	0.04	13	45168	0.12

Note: SD display the standard deviations of log(wage03), the logarithms of 2003 annual salary of men or women within strata. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

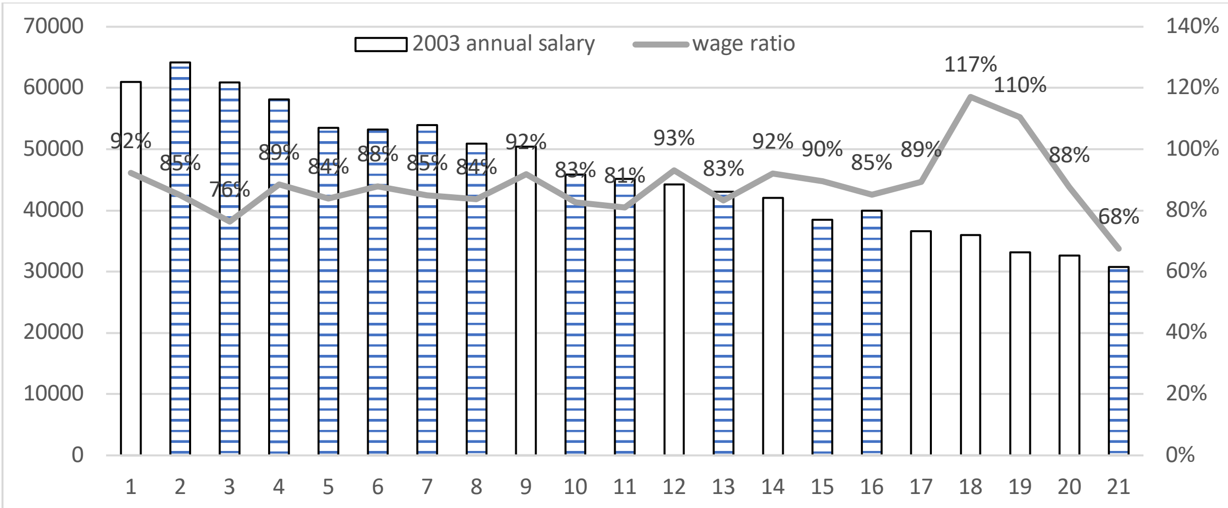

Figure 2. 2003 Annual Salary and Wage Ratio across Strata
(Pattern filled bars mean gender wage gap is statistically significant.)

To sum up, first of all, after "equalizing" the joint distribution of the observed productivity-related characteristics, the adjusted women-to-men wage ratio is 87.4%. Compared with the unadjusted women-to-men wage ratio (70.78%), the adjusted wage ratio is a lot larger yet still significantly different than 100%. However, the fact that it is not close to 100% suggests that the observed productivity-related characteristics cannot explain all the gender wage gap. From stratum 1 to stratum 21, when the "profile" of observed productivity-related characteristics is more likely to be women's, the annual salary is lower while the major GPA is higher. However, the gender wage gap is universal in all strata and is statistically significant different than zero in majority of the strata.

6. Conclusion

This study was intended to find evidence on whether female college graduates still earn less than their male counterparts after their profiles of productivity-related characteristics are statistically adjusted to be equivalent. Making use of the rich information provided by the 1993 cohort of the Baccalaureate and Beyond Longitudinal Study, this study combines the propensity score stratification and the Blinder-Oaxaca conceptual framework to measure the gender wage gap among this college graduated population. The empirical evidence gathered from this study shows that the observed productivity-related characteristics cannot completely explain the gender wage gap. In other words, a discriminatory gender wage gap might still exist among these college graduates in the labor market.

The analytic results show that, after statistically adjusting for all the observed productivity-related characteristics, the women-to-men wage ratio is 87.4% and statistically significant different from 100%. Compared with the unadjusted women-to-men wage ratio in the descriptive analysis (70.78%), the adjusted wage ratio is a lot larger. However, the fact that it is statistically lower than 100% suggests that the observed productivity-related characteristics cannot completely explain the gender wage gap. Moreover, gender difference in psychological traits and personal choices based on job preferences are unobserved in the data, which may explain at least part of the gender wage gap. Hence, the estimated difference between the adjusted and unadjusted wage ratio may be viewed as a conservative estimation of the discriminatory gender wage gap, before we can include and account for more explanatory variables in further analysis.

Additional analysis for sub-sample populations (in Appendix) show that, gender wage gap exists but is not statistically significant different from zero among people in minority racial/ethnical groups. However, first-generation college students or students coming from low-income families have the same level, if not larger, of gender wage gap as the whole population. When looking at the college graduates' majors, while high GPA does not necessarily lead to the highest annual salary, students with high major GPA (>3.5) have the smallest gender wage gap compared with other two GPA groups. The gender wage gap among people who majored in

STEM is not significantly different than zero, and they have the highest average annual salary, which suggests that majoring in STEM might help narrow the gender wage gap. However, if women are underrepresented in STEM majors and furthermore, in STEM occupations (they are underrepresented in reality), this might convert into women's disadvantages in labor market.

Besides analyzing certain sub-populations, propensity score stratification allows us to observe the gender difference throughout the joint distribution of the observed explanatory variables. When the "profile" of productivity-related characteristics is more likely to be men's, the annual salary is higher while the major GPA is lower. However, gender wage gap universally exists in all strata and is statistically significant different than zero in majority of them. Therefore, the gender wage gap is driven by both the within-stratum and across-stratum difference in annual salary.

Applying the new method provides valuable evidence that has not been found before. However, this study also has its limitations. First, due to the data limitation of the B&B study, that is, no information on the non-traditional explanatory factors of the wage such as the psychological traits and preferences, this study can only provide a conservative estimation of the gender discrimination possibly existing in the labor market. To get a more accurate estimation, similar analysis might need to be carried out using other survey data that ask questions about decision process and self-reported psychological traits. Second, 2403 subjects were excluded from the analytic sample because they did not report positive annual salary in 2003. 206 subjects have a predicted propensity score beyond the common support of women and men sub-samples. A supplementary analysis may investigate, within each gender group, whether those whose outcome was missing (mostly due to not participating in the labor force) tend to have higher or lower potential earnings than those whose outcome was observed.

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Appendix

1. Detailed derivation of the BO decomposition

The Blinder-Oaxaca (BO) decomposition was proposed by Oaxaca (1973) and Blinder (1973) to decompose the wage differential for two exclusive demographic groups, such as men and women, white and non-white. The BO method is based on a semi-linear wage equations which is applicable to each person of the two groups:

$$\ln(W_i) = X_i{}'\beta + u_i \tag{1}$$

i=1,2,3,…, n

W_i : The wage (hourly wage if available) for individual i

X_i : a vector of the productivity-related characteristics for individual i

β : a vector of coefficients of the productivity-related characteristics

u_i : the disturbance term, where $u_i \sim N(0, \sigma^2)$

For gender groups,

$$\ln(\overline{W}_n) = \bar{X}_n{}'\hat{\beta}_n \tag{2}$$

Subscript n=m (men) or f (women);

$\overline{W}_n$: the mean of wage within the certain gender group;

$\bar{X}_n$: the mean of the productivity-related characteristics within the certain gender group;

$\hat{\beta}_n$: the estimated coefficients of the wage function within the certain gender group.

The wage differential between two gender groups is defined as:

$$G = \frac{\bar{W}_m - \bar{W}_f}{\bar{W}_f} = \frac{\bar{W}_m}{\bar{W}_f} - 1 \tag{3}$$

which is equivalent to,

$$\ln(G+1) = \ln\left(\frac{\bar{W}_m}{\bar{W}_f}\right) = \ln \bar{W}_m - \ln \bar{W}_f \tag{4}$$

The group average equivalence of (1) is $\ln(\bar{W}_n) = \bar{X}_n{}'\hat{\beta}_n$, hence,

$$\ln(G+1) = \bar{X}_m{}'\hat{\beta}_m - \bar{X}_f{}'\hat{\beta}_f \tag{5}$$

Use the women's wage as the reference, which assumes that given the same productivity-related characteristics, men will have the same wage level as the one women currently have if the labor market does not have gender-based discrimination.

Let

$$\Delta\bar{X}' = \bar{X}_m{}' - \bar{X}_f{}'$$

$$\Delta\hat{\beta} = \hat{\beta}_m - \hat{\beta}_f$$

$\Delta\bar{X}'$: the vector of productivity-related characteristics difference between the average of the two gender groups, e.g. one element of the vector is the difference of the average education level between male and female workers

$\Delta\hat{\beta}$: the difference of the estimated coefficients of productivity-related characteristics cs between the two gender groups, e.g. hold all else equal, if men with a certain education level are paid more than the women with the same education level, this term will be positive. If discrimination does not exist in the labor market, then the productivity-related characteristics of both gender groups are paid in the same way, this term will be zero.

Add and then subtract a counterfactual term, $\bar{X}_m{}'\hat{\beta}_f$

$$\ln(G+1) = \bar{X}_m{}'\hat{\beta}_m - \bar{X}_m{}'\hat{\beta}_f + \bar{X}_m{}'\hat{\beta}_f - \bar{X}_f{}'\hat{\beta}_f \tag{6}$$

The counterfactual term $\bar{X}_m{}'\hat{\beta}_f$ is the product of the productivity-related characteristics of men, $\bar{X}_m'$, and the estimated coefficients of productivity-related characteristics of women, $\hat{\beta}_f$, which is assumed to be the coefficients of a wage function in the absence of discrimination. In other words, this is the counterfactual wage men would receive if the gender changes, holding all the productivity-related characteristics constant.

The above equation is equivalent to,

$$\ln(G+1) = \bar{X}_m'\Delta\hat{\beta} + \Delta\bar{X}'\hat{\beta}_f \tag{7}$$

The first term $\bar{X}_m'\Delta\hat{\beta}$ is the product of the mean of productivity-related characteristics of men, $\bar{X}_m'$, and the difference between the estimated coefficients of productivity-related characteristics between the two gender groups, $\Delta\hat{\beta}$. This term represents the part of the wage the men receive which attributes to the different paid-off of the productivity-related characteristics in the labor market. The sign of this term depends on the sign of $\Delta\hat{\beta}$. That is, if the labor market is favored to men, this term is positive, vice versa. If it's zero, it means there is no gender-based discrimination in the labor market.

The second term $\Delta\bar{X}'\hat{\beta}_f$ is the product of the vector of productivity-related characteristics difference between two gender groups, $\Delta\bar{X}'$, and the estimated coefficients of the productivity-related characteristics of the female, $\hat{\beta}_f$, which is assumed to be the coefficients of a wage function in the absence of discrimination. Hence, this term represents the part of wage differential that is attributable to the difference of productivity-related characteristics between the two gender groups, $\Delta\bar{X}'$, if the wage is purely determined by them in the absence of discrimination.

The measurement of discrimination is defined as:

$$D = \frac{\frac{W_m}{W_f} - (\frac{W_m}{W_f})^0}{(\frac{W_m}{W_f})^0} = \frac{\frac{W_m}{W_f}}{(\frac{W_m}{W_f})^0} - 1 \tag{8}$$

$\frac{W_m}{W_f}$: the man-to-woman wage ratio observed in the labor market, which is equivalent to $\frac{\bar{W}_m}{\bar{W}_f}$ for the perspective of group-level wage ratio

$(\frac{W_m}{W_f})^0$: the man-to-woman wage ratio in the absence of discrimination, which is equivalent to $(\frac{\bar{W}_m}{\bar{W}_f})^0$ for the perspective of group-level wage ratio

Its equivalence in natural logarithm is,

$$\ln(D+1) = \ln\left(\frac{\bar{W}_m}{\bar{W}_f}\right) - \ln\left(\left(\frac{\bar{W}_m}{\bar{W}_f}\right)^0\right) \tag{9}$$

From (4), we know that, $\ln(G+1) = \ln\left(\frac{\bar{W}_m}{\bar{W}_f}\right)$, hence,

$$\ln(G+1) = \ln(D+1) + \ln\left(\left(\frac{\bar{W}_m}{\bar{W}_f}\right)^0\right) \tag{10}$$

From (7), we know that $\ln(G+1) = \bar{X}_m'\Delta\hat{\beta} + \Delta\bar{X}'\hat{\beta}_f$, where $\bar{X}_m'\Delta\hat{\beta}$ represents the part of wage differential due to the different paid-off of productivity-related characteristics between men and women; $\Delta\bar{X}'\hat{\beta}_f$ represents the part of the wage differential due to average difference in productivity-related characteristics between the gender groups in the absence of gender-based discrimination.

Also,

$$\ln\left(\left(\frac{\overline{W}_m}{\overline{W}_f}\right)^0\right) = \ln(\overline{W}_m^0) - \ln\left(\overline{W}_f^0\right) = \bar{X}_m{}'\hat{\beta}_f - \bar{X}_f{}'\hat{\beta}_f = \Delta\bar{X}'\hat{\beta}_f \tag{11}$$

where $\hat{\beta}_f$ is the coefficients of the productivity-related characteristics in the absence of discrimination under the assumption.

From (7), (10) and (11) we know that,

$$\ln(D+1) = \bar{X}_m'\Delta\hat{\beta} + \Delta\bar{X}'\hat{\beta}_f - \Delta\bar{X}'\hat{\beta}_f = \bar{X}_m'\Delta\hat{\beta} \tag{12}$$

Equations (10) to (12) further show the decomposition of the wage differential can be expressed in different ways but still has the same meanings. The wage ratio in the absence of gender-based discrimination, $\ln\left(\left(\frac{\overline{W}_m}{\overline{W}_f}\right)^0\right)$ corresponds to the part that can purely explained by the observed intergroup difference, $\Delta\bar{X}'\hat{\beta}_f$. The measurement of discrimination, $\ln(D+1)$ corresponds to the part that attributes to the gender difference in characteristics paid-off, $\bar{X}_m'\Delta\hat{\beta}$. The former is often called the explained part of the gender wage differential, and the latter is called the unexplained part. Here, “explained” means that can be explained by the observed productivity-related characteristics.

It is clear that the original decomposition method proposed by Oaxaca and Blinder only accounts for the wage differential in the level of group averages. It is not informative on the wage distribution within each group and the difference of the wage distribution between groups. To overcome the informative limitation on distribution, later researchers proposed to use the quintile regression on the wage equations(Juhn et al., 1993).This type of method replaces the logarithm of wage with the quantiles of wage as the outcome variables so that the distribution of wage will be included when estimating the wage function. The results presents the decomposition the gender wage differential at selected quantiles, such as the three quartiles, to show the level of gender-based discrimination for people earning different level of wage.

2. Generalized BO Decomposition in the Application of Gender Wage Gap

The application of this generalized BO decomposition theory to the case of gender wage gap is as followed. Denote gender as N∈ $\{f, m\}$, earnings as W, the productivity-related characteristics as vector X. Let $g(X|N)$ denotes the density function of the productivity-related characteristics given gender, the expected earning function given gender is given by:

$$E[W|N] = \int E[W|N, X]g(X|N)dX$$

Let

$$E[W|N = f] \equiv E_f[W]$$

$$E[W|N = m] \equiv E_m[W]$$

Then the expected wage differential between two genders is:

$$\Delta \equiv E_m[W] - E_f[W]$$

Let

$$g(X|N = f) \equiv g_f(X)$$

$$g(X|N = m) \equiv g_m(X)$$

$$E_f[W] = \int E_f[W|X]g_f(X)dX$$

$$E_m[W] = \int E_m[W|X]g_m(X)dX$$

$g_f(X)$ can be interpreted as the density function of vector X, which represents the distribution of the productivity-related characteristics of the female group in the labor market, which corresponds to $\bar{X}_f'$ in the BO decomposition. $E_f[W]$ is the expected wage of an employee given the gender, which can be viewed as the non-parametric alternative of the wage function, corresponding to $\hat{\beta}_m$ in the BO decomposition. $g_m(X), E_m[W]$ are the corresponded terms for male.

Hence,

$$\Delta = \int E_m[W|X]g_m(X)dX - \int E_f[W|X]g_f(X)dX$$

$$= \int E_m[W|X]g_m(X)dX - \int E_f[W|X]g_m(X)dX$$

$$+ \int E_f[W|X]g_m(X)dX - \int E_f[W|X]g_f(X)dX$$

$$= \int (E_m[W|X] - E_f[W|X])g_m(X)dX + \int E_f[W|X](g_m(X) - g_f(X))dX$$

$\int E_f[W|X]g_m(X)dX$ is a counterfactual term, which can be interpreted as the expected wage of the male in the labor market using the women's wage function. It can also be viewed as the expected wage of the female if they had the male's distribution of productivity-related characteristics, X.

$E_m[W|X] - E_f[W|X]$ represents the difference of the expected wages for male and female given the productivity-related characteristics in the non-parametric set-up. Consequently, the first term corresponds to the $\bar{X}'_m \Delta\hat{\beta}$ in BO decomposition, which implies the measurement of discrimination. $g_m(X) - g_f(X)$ represents the difference between the density functions of productivity-related characteristics for male and female in the labor market. Hence, the second term corresponds to $\Delta\bar{X}'\hat{\beta}_f$ in BO decomposition, which indicates the part of wage differential that can be explained by the difference in the distribution of productivity-related characteristics

The counterfactual term can be generated by reweighting the distribution of one group to that of another one using a weight as:

$$w(X) = \frac{\Pr\,(X|N = m)}{\Pr\,(X|N = f)}$$

That is,

$$g_m(X) = w(X) g_f(X)$$

The counterfactual term is:

$$\int E_f[W|X] g_m(X) dX = \int w(X) E_f[W|X] g_f(X) dX$$

The denominator and numerator of the weighting function can be transformed by Bayes' rule and the weight function is equivalent to:

$$w(X) = \frac{\Pr(N = m|X)\,/\Pr\,(N = m)}{\Pr(N = f|X)\,/\Pr\,(N = f)}$$

which is the inverse of the weights used in the propensity-score based method of inverse-probability-of-treatment-weighting (IPTW). Nevertheless, in the case of gender wage gap, it is inapplicable to use this weighting method as there are more than one variables of productivity-related characteristics in X, a new decomposition method based on the basic concept of propensity score is proposed in the following sub-section.

3. Basic concept of propensity score

The concept of propensity score was first proposed by Rosenbaum and Rubin (1983) to remove the bias between treatment and control groups that are due to the observed covariates. Since then, propensity score based methods are broadly used to estimate causal effects of treatment in nonrandom experimental setting, such as social experiments and observational studies. The basic concept and assumption of propensity score is as followed.

Denote Z as the binary variable indicating the assignment of treatment. Z=0 indicates being assigned to the control group; Z=0 indicates being in the treatment group. Propensity score is defined as the probability of being assigned to treatment group given the observed covariates, X.

$$e(X) \equiv \Pr\,(Z = 1|X)$$

The propensity score is a balancing score. The balancing score is a function of the observed covariates X with the property of "balancing" the conditional distribution of the treatment and control groups. That is, conditioning on the balancing score, the distributions of the observed covariates X are the same for treatment and control groups. Hence, given the propensity score, the distribution of the observed covariates X are independent to the treatment assignment:

$$X \perp Z \mid e(X)$$

Propensity score based methods is based on the conditional ignorability assumption, where the treatment assignment is strongly ignorable given the observed covariates X, that is, the potential outcome is independent to treatment assignment given X:

$$Y \perp Z \mid X$$

Denote Y_1 as the outcome of the treated, Y_0 as that of the control, the average treatment effect of the sample population is:

$$ATE = E_x[(E(Y_1|X) - E(Y_0|X)] = \int [(E(Y_1|X) - E(Y_0|X)]dX$$

In Rosenbaum and Rubin (1983), it is proved that if the above assumption holds, then the treatment assignment is also strongly ignorable given the propensity score, that is,

$$Y \perp Z \mid e(X)$$

Hence, at any certain value of the propensity score, the difference between the treatment and control groups is an unbiased estimate of the treatment effect for the people with this value of propensity score. The average treatment effect of the sample population is:

$$ATE = E_{e(x)}\{[E[Y_1|e(X)] - E[Y_0|e(X)]\} = \int [E[Y_1|e(X)] - E[Y_0|e(X)]\, de(X)$$

With the above property, the propensity score can be used in sample matching, stratification and covariate adjustment.