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A Quantitative Study of Mathematical Language in Classrooms

Zachary Himmelsbach
Harvard University

Heather C. Hill
Harvard University

Jing Liu
University of Maryland College Park

Dorottya Demszky
Stanford University

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A Quantitative Study of Mathematical Language in Classrooms

Zachary Himmelsbach*
Harvard University
719 Washington St. #201
Boston, MA 02124
zah972@g.harvard.edu

Heather C. Hill
Harvard University
Gutman Library, Room 445
6 Appian Way | Cambridge, MA 02138
heather_hill@harvard.edu

Jing Liu
University of Maryland College Park
2203 Benjamin Building
University of Maryland, MD 20740
jliu28@umd.edu

Dorottya Demszky
Stanford University
CERAS Bldg, 520 Galvez Mall
Stanford, CA 94305
ddemszky@stanford.edu

*Corresponding Author

Abstract

This study provides the first large-scale quantitative exploration of mathematical language use in U.S. classrooms. Our approach employs natural language processing techniques to describe variation in the use of mathematical language in 1,657 fourth and fifth grade lessons by teachers and students in 317 classrooms in four districts over three years. Students' exposure to mathematical language varies substantially across lessons and between teachers. Students whose teachers use more mathematical language are more likely to use it themselves, and they perform better on standardized tests. These findings suggest that teachers play a substantial role in students' mathematical language use.

Introduction

To attain proficiency in mathematics, students must be able to understand and use mathematical language. Differing from colloquial speech and writing, mathematical language contains specialized vocabulary with precise meanings (Pimm, 1987; Schleppegrell, 2007). Teachers seeking student mastery of mathematical language undertake several tasks: they must connect mathematical terms to the mathematical content and representations those terms signify; they must model the use of mathematical terms in their instruction; and they must encourage students to practice using the terms as they speak about mathematics (Hughes, Powell & Stevens 2016). Teachers vary in their engagement with these teaching tasks (Hughes, Powell & Stevens 2016; Lane, O'Meara & Walsh, 2019; Ernst-Slavit & Mason, 2011). Some prefer to use colloquial language in place of mathematical terms, believing it enhances students' understanding (Hughes, Powell & Stevens 2016). Other teachers' usage of mathematical terms may be limited by their own lack of fluency (e.g., Gürefe, 2018).

The study of mathematical language in classrooms has been largely limited to theoretical inquiry and case studies. Taking advantage of techniques from natural language processing (NLP), we undertook a large-scale study of how teachers' and students' use of mathematical vocabulary varies across classrooms. Our exploration of mathematical language takes up the following questions:

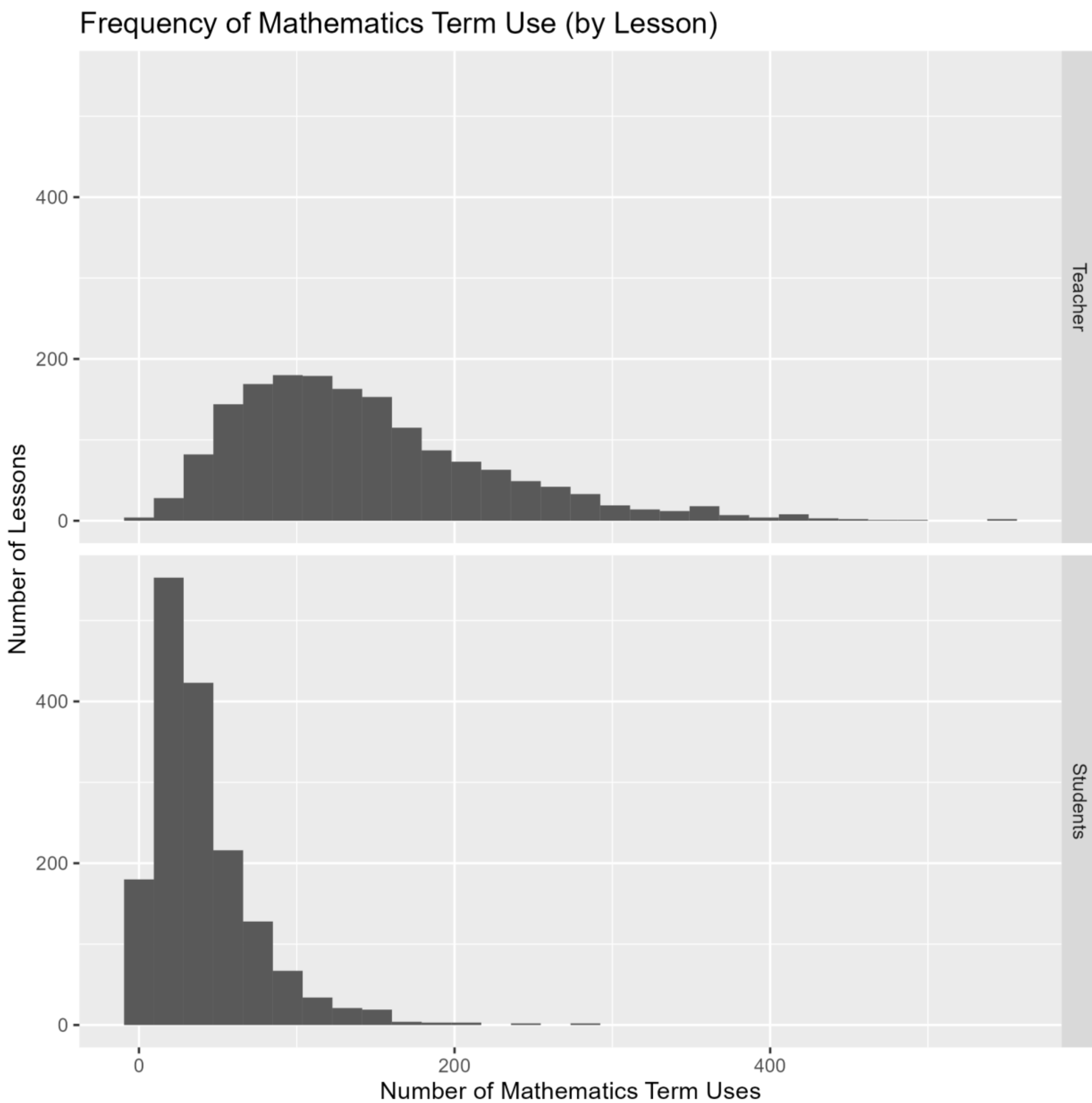
RQ 1: How frequently do a sample of upper-elementary teachers and students use mathematical terms? How much does this vary across lessons?
RQ 2: How much do these teachers vary in their use of mathematical terms? Are such differences explained by teacher background or mathematical knowledge?
RQ 3: Does a teacher's use of mathematical terms predict their value-added scores and students' use of the same mathematical vocabulary?

We answer these questions using anonymized transcripts from 1,657 4th-5th grade math lessons taught by 317 teachers over three years. The National Center for Teacher Effectiveness (NCTE) collected these data between 2010-2013 across four school districts that largely serve historically marginalized students (Demszky & Hill, 2022). The dataset also includes information on teachers' mathematical coursework, assessment of their mathematical knowledge for teaching (MKT), and teacher-by-year value-added scores based on students' standardized test performance. Students were assigned to teachers as per typical school policy in the first two years of the study and randomly assigned in the third year.

Using NLP techniques, we created a dictionary of mathematical terms and extracted those terms from transcripts. See supplemental materials for details.

Results

RQ1: Teacher and student use of mathematical terms



Teachers used mathematical terms 127 times in the median lesson, but frequency varied substantially across lessons (sd = 82). This variation precipitates large differences: over the course of a school year (160 lessons), students in classrooms at the 75th percentile of this measure hear an estimated 10,000 more examples of mathematical vocabulary than their peers in classrooms at the 25th percentile.

Lessons also differed in the amount of mathematical vocabulary used by students. In the median lesson, a teacher heard 38 uses of mathematical terms from their students. As above, this varies, with lessons at the 25th and 75th percentiles eliciting 26 and 50 math terms, respectively. Mathematical terms make up a greater proportion of student speech (5.7% vs teachers' 2.8%)

RQ 2: Variation in the use of mathematical terms among teachers

Next, we examined the extent to which teachers in our sample varied systematically in using mathematical terms. The average teacher used 140 mathematics terms per lesson (sd 53.5). Variance decomposition reveals persistent teacher differences, with teachers accounting for 18% of the variance in the mathematical language observed at the lesson level ($p < 0.01$). There appears to be few differences in the use of mathematical terms at the school level. The high level of residual variance (82%) relates to differences within teachers (e.g., based on the content of lessons).

Of the observed teacher characteristics, a joint regression model finds that only MKT assessment scores significantly predict more frequent use of mathematical vocabulary (p <

0.01). Estimates for teachers' possession of a math major/minor or their number of college math courses taken were too imprecise to draw conclusions. See details in supplementary materials.

RQ 3: Connection between teachers' use of mathematical terms and student outcomes

Teachers who more frequently exposed their students to mathematical vocabulary tended to have higher value-added scores. We conducted two analyses that support this claim. First, we calculated teachers' average use of mathematical terms in each year of the NCTE study and estimated the correlation between this measure and teachers' value-added scores within the same year. Value-added scores adjust for differences in students' demographics and prior achievement. The estimated correlation is positive (.093, $p < .10$). One hundred more mathematical terms per lesson is associated with a 0.03 standard deviation difference in value-added scores.

To overcome possible sorting of students to teachers, and reflection issues (i.e. students' use of terms encouraging teachers to use additional math terms and vice versa), we then used the amount of mathematical vocabulary that teachers elicited from their students in the non-randomization years of the NCTE study to proxy for teachers' propensity to elicit math terms from their students in the randomization year. We found that teachers whose past students used more mathematical vocabulary caused their randomization-year students to use more mathematical terms ($\rho = .29$, $p < 0.01$). This suggests teachers play a systematic role in students' uptake of mathematical language.

Discussion

Teachers vary in the frequency with which they use mathematical terms within lessons, and this variation predicts student growth on standardized tests. Students' use of mathematical terms was far less frequent but appears to correlate with the frequency of mathematical terms in their teacher's speech. Together, these findings suggest that we should develop and evaluate interventions that encourage teacher and student use of mathematical vocabulary in everyday instruction.

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Supplemental Materials: Methodology

To create an initial dictionary of mathematical terms, we scraped publicly available glossaries of mathematical terms from two K-6 math curricula, Zearn and Illustrated Mathematics. A small number of terms, “row” for example, were dropped because they have common, non-mathematical meanings in classrooms. Some terms were stemmed to match multiple forms of the word (e.g. “multiplication” is stemmed to “multipl” to also match “multiply” or “multiples”). Initialisms, like “LCF”, were supplemented with their expanded forms. So, for example, both “LCF” and “least common factor” are included. In the end, we generated a list of 256 mathematical terms. Of these, 224 terms appear at least once in the NCTE transcript data. Many of the words that never appear are statistics terms from the 6th grade curriculum glossaries. Our transcripts come from 4th and 5th grade classrooms, so this makes sense.

We took several steps to ensure our list represents the set of terms used in 4th and 5th grade classrooms. First, we generated word2vec embeddings for all words in our corpus of transcripts. We then calculated pairwise cosine similarities for the embeddings of our list of mathematical terms with all other words in the corpus. We would expect similar words - i.e. other mathematical words not in our initial list - to have high cosine similarities with at least one term in our list. When we ranked words by their maximum cosine similarity with a math term, we didn’t find any mathematical terms among the top-ranked words. We note that this method only identifies unigrams (single words), not multi-word expressions or phrases.

To estimate how many terms we might have missed, we audited a random sample of teacher utterances from the transcripts. We stratified our sample on whether the utterance contained at least one mathematical term in our list. The sample included 10,000 utterances (out of about 580,000 total in the corpus), with half of them containing at least one mathematical term from our dictionary. The stratification was motivated by the idea that utterances with some mathematical terms might be more likely to contain additional ones, because we know those utterances are math-related (as opposed to classroom management focused). We found a single mathematical term in this sample that was not included in our list. This term was not used in any other utterance in the dataset. Given the results of this audit, we estimate that we have missed a very small number of used mathematical terms. In addition, any terms we have missed are likely to be rarely used.

With this list, we calculated the number of unique mathematical terms in each utterance. This choice means that repeating the same term multiple times within an utterance will have the same weight in our measures as using it a single time. From these utterance-level counts, we constructed a set of lesson-level measures of mathematical language. We do this separately for teacher and student speech.

Our primary measure of mathematical language is a simple sum of the utterance-level counts. We also constructed a normalized measure: the number of mathematical terms used divided by the total number of words spoken in the lesson. Additionally, we constructed an H-index based measure. None of our results varied substantively across these measures, so we report results based on the simple measure for ease of interpretability. To produce a teacher-level measure, we averaged their count of mathematical terms used over all their lessons.

Variance Model for RQ 2

$$MathTerms_{isd} = \lambda_d + \mu_i + \zeta_s + \varepsilon_{isd}$$

$$\mu_i \sim N(0, \sigma_i^2)$$

$$\zeta_s \sim N(0, \sigma_s^2)$$

$$\lambda_d \sim N(0, \sigma_d^2)$$

Here, teachers i, are nested within schools s. We also include fixed effects d for the districts (to adjust for differences in curricular materials). When we exclude these fixed effects, the estimated variances do not meaningfully change. The estimates of the variance accounted for by each level differs by less than 1.5 percentage points between these models.

Supplemental Table 1: Relationship Between Observed Teacher Characteristics and Mathematical Term Use

	Dependent variable:
	Mathematical Terms Used (Per Lesson)
MKT SCORE	9.078***
	(3.459)
MATH MAJOR	11.959
	(11.839)
MATH CONTENT	-5.980
	(3.811)
MATH COURSES	-0.049
	(0.034)
Constant	164.656***
	(10.019)
Observations	834
R^2	0.015
Adjusted R^2	0.010
Residual Std. Error	89.350 (df = 829)
F Statistic	3.178** (df = 4; 829)
Note:	*p**p***p<0.01