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EdWorkingPaper No. 24-1047

How and Why Racial Isolation Affects Education Costs & the Provision of Equal Educational Opportunity

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This article provides a review of prior empirical work exploring whether and to what extent school district racial composition affects the costs associated with providing equal educational opportunity to achieve a common set of outcomes. This prior work mainly involves education cost function modeling, on several specific states and in an earlier version of our national education cost model. Here, we update the national education cost model and apply a series of tests for selecting the optimal cost model and determining a) whether it is necessary to retain measures of racial composition in the model and b) the effect those measures have on the estimated costs to achieve common outcomes. We find that the optimal model includes an interaction term between % enrollment that is black and population density and that for majority Black enrollment urban districts, the predicted costs per pupil are 20 to 50% higher when using models with this measure than when using models with race neutral alternatives. While changes in cost estimates for these districts are large, aggregate national cost increases from including racial composition are 1.3 to 2.7% in most years.

VERSION: September 2024

Suggested citation: Baker, Bruce D.. (2024). How and Why Racial Isolation Affects Education Costs & the Provision of Equal Educational Opportunity. (EdWorkingPaper: 24-1047). Retrieved from Annenberg Institute at Brown University: https://doi.org/10.26300/fy6k-4v18

How and Why Racial Isolation Affects Education Costs & the Provision of Equal Educational Opportunity

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Draft date: September 20, 2024

Abstract

This article provides a review of prior empirical work exploring whether and to what extent school district racial composition affects the costs associated with providing equal educational opportunity to achieve a common set of outcomes. This prior work mainly involves education cost function modeling on several states and in an earlier version of our national education cost model. Here, I update the national education cost model and apply a series of tests for selecting the optimal cost model and determining a) whether it is necessary to retain measures of racial composition in the model and b) the effect those measures have on the estimated costs to achieve common outcomes. We find that the optimal model includes an interaction term between % enrollment that is Black and population density and that for majority Black enrollment urban districts, the predicted costs per pupil are 20 to 50% higher when using models with this measure than when using models with race neutral alternatives. While changes in cost estimates for these districts are large, aggregate national cost increases from including racial composition are 1.3 to 2.7% in most years.

Introduction

On February 6 2007, an Associated Press headline in the St. Louis Post-Dispatch read, "More Should Be Spent on Black Students, Experts Say". The article was referring to testimony given a day earlier in a case challenging Missouri's school funding formula, where two separate experts, myself included, had conducted cost modeling to identify the costs of achieving specific outcome goals for Missouri children. In separate, independent reports, William Duncombe and I had found the inclusion of district racial composition to be an inescapable factor in estimating those costs – that is, that costs were necessarily higher in districts serving larger Black student enrollment shares.

Researchers have often found racial achievement gaps that persist above and beyond differences in economic status or parental/family educational background (Reardon, Robinson-Cimpian & Weathers, 2014; Barton & Coley, 2010; Baker, Keller-Wolff & Wolf-Wendel, 2000). Some have found that specific interventions or reforms, including class size reduction have differential positive effects on student outcomes by their race (Krueger & Whitmore, 2001; Levin, Belfield, Muennig & Rouse, 2007). Still others have found that choices by teachers of where they'd rather teach or stay are influenced by the student racial composition of schools, resulting in the need for a wage premium to retain teachers in schools with large Black student populations (Hanushek, Kain & Rivkin, 2004; Hanushek & Rivkin, 2006). The role of race, racial segregation, isolation, driven by persistent racial discrimination in schooling, housing and beyond creates undeniable racial inequalities throughout American education systems, realized in nearly any or all data and measures on those systems, from outcomes, to contexts and resources (Baker, Di Carlo & Green, 2022).

Thus, one would expect racial segregation and isolation to affect the costs of providing equal educational opportunity through a variety of interrelated mechanisms. It stands to reason, for example, that if it takes a higher wage to recruit and retain teachers of similar qualifications in schools with larger Black enrollment shares, that it would cost more just to provide equal quality inputs to schooling (comparable teachers and class sizes) in majority Black versus majority white schools. Further, it stands to reason that if scholars have found that class size reduction has differential effects by race – yielding larger advantages for Black than white students – and that targeted class size reduction can be an effective strategy for reducing Black-white achievement gaps, that the overall costs (teacher wage x teacher quantities) to achieve common outcome targets may differ in majority Black versus majority white schools, all else equal.

On the one hand, eliminating racial segregation and isolation itself can reduce racial inequality in student outcomes, because it mitigates various detrimental effects of racial isolation (Johnson, 2019). Where racial segregation and isolation and their lingering effects do persist – which is nearly everywhere in the U.S. – other policy leverage is required (Baker, Di Carlo & Green, 2022). Most often, however, that "other" policy leverage intentionally avoids the role of race, racism and racial discrimination in causing the racial disparities which are deeply intertwined with economic and educational disparities, yielding incomplete, imprecise remedies (Baker & Castillo, 2024).

At the height of the school desegregation legal and policy movement in the post-Brown era, there emerged a parallel path toward mitigating schooling inequality – a legal path that took great pains to identify *anything but race* that might be an objectionable cause of educational inequality. Federal, then state constitutional challenges to the equity and adequacy of school funding rarely made explicit mention of racial disparities, often leaning in to alternative measures and explanations for the disparities including differences in taxable property wealth and income of households and asserting those disparities should receive the same heightened scrutiny as racial disparities (Heise, 1995; Ryan, 1999).

Wealth and income related school funding disparities were and still are highly racially correlated, because in many cases they were caused by racial discrimination regarding who could live where, coupled with access to employment (Baker, Di Carlo & Green, 2022). Only a handful of challenges to school funding formulas have explicitly addressed racial disparities and even fewer included legal claims related to racial disparities.[1] As such, little empirical research designed to support school funding remedies and state school finance legislation has directly addressed racial disparities or how to mitigate them. A surprisingly modest body of empirical literature even explores, quantifies and decomposes racial disparities in school funding and causes of those disparities (Baker et al., 2020; Bifulco, 2005; Bifulco & Souders, 2023; Sosina & Weathers, 2019 and Weathers & Sosina, 2022).

Race-avoidant challenges to school funding inequality evolved from the 1970s to present day in state courts, under state constitutional requirements while challenges to racial segregation came and went primarily in federal courts from the immediate post-Brown era through the mid-2000s when the Supreme Court eventually declared that existing within district integration policies (in Seattle and Louisville) could no-longer survive strict scrutiny.[2] Even when both types

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[1] Specifically, *Robinson v. Kansas*, 295 F.3d 1183 (10th Cir. 2002), *Powell v. Ridge*, 247 F.3d 520 (3d Cir. 2001) & *African American Legal Defense v. New York State*, 8 F. Supp. 2d 330 (S.D.N.Y. 1998) in Federal Courts and among the causes of action in *Montoy v. State*, 279 Kan. 817, 112 P.3d 923 (2005).

[2] *Parents Involved in Community Schools v. Seattle School District, No. 1*, 426 F.3d 1162 (9th Cir. 2005) & *Meredith v. Jefferson County Board of Education*, 548 U.S. 938 (2006).

of legal challenges occurred simultaneously in state courts, as in Connecticut, the legal challenges and remedies occurred independent of and without regard for one another, albeit competing for resources.[3]

Baker, Green and Oluwole (2008) explained that, given what Duncombe (2007) had found in Missouri a year earlier, achieving equal educational opportunity through school finance reform would necessarily require consideration of race. That, if it costs more to achieve state mandated outcome goals as a function of district racial composition, above and beyond other factors including poverty concentration, shares of non-English speaking students and other cost drivers, those costs must be considered in the design of school funding remedies. They must be weighted as a "plus factor" in reform legislation. Not doing so is to knowingly deprive Black students in racially isolated districts of equal educational opportunity.

This article explores and expands on the limited literature that attempts to identify and quantify the role that race, racial segregation and racial isolation play in determining the costs of providing equal educational opportunity. Equal educational opportunity, as described by Baker and Green (2012) requires that all children, whatever their own backgrounds or wherever they attend school are provided equal opportunity to achieve common outcome goals. Part of ensuring equal educational opportunity is ensuring that children have access to the resources (class sizes, teacher quality, etc.) needed to have equal opportunity to achieve those outcomes, which in turn

[3] Sheff vs. O'Neill was a desegregation case brought in the 1990s in Connecticut (*Sheff v. o'Neill*, 238 Conn. 1, 678 A.2d 1267 (1996). That case led to the adoption of interdistrict choice, magnet school programs in Hartford, CT which was accompanied by significant state funding to support the magnet school program. In the decades that followed, the state's general school aid formula was challenged separately in CJEFF v. Rell (*CONNECTICUT COALITION FOR JUSTICE v. Rell*, 176 A.3d 28, 327 Conn. 650 (2018), where the state's high court upheld the school aid formula.

requires appropriately calibrated financing. Of particular interest are three major questions, which have only sparsely been addressed in existing literature:

1. What evidence exists that district or school racial composition influences the costs of providing equal educational opportunity?
2. What other research evidence supports causes or reasons why the costs of providing equal educational opportunity vary by school or district racial composition?
3. What empirical evidence exists regarding integration and/or consolidation of school districts by race and effects on costs and costs variation? That is, does integration reduce or otherwise change the distribution of costs associated with racial isolation?

We revisit these questions now for a variety of reasons. Baker, Green and colleagues addressed the importance of race in determining education costs and constitutional questions surrounding the use of race factors in state school finance formulas in a series of articles between 2008 and 2011 (Baker, 2011, Baker & Green, 2009; Green, Baker & Oluwole, 2008). Those articles followed a pair of U.S. Supreme Court rulings in which the court disallowed consideration of race in school assignment programs (for integration purposes). We argued at that time, that if schools were to remain racially isolated which they likely would, one means of providing equal educational opportunity would be consideration of the costs imposed by racial isolation (Green, Baker & Oluwole, 2008). I revisit this question now because more recent articles and reports presents the case that such factors can and should play a role in the provision of reparations to Black communities for the educational damages imposed by racial residential housing discrimination (Baker, Green & Di Carlo, 2022; Green, Baker & Oluwole, 2020). Further, as illustrated in a more recent article, nationally equated state assessment data from the Stanford Education Data Archive, along with other national data updates and improvements make it possible for us to estimate a National Education Cost Model (Baker, Weber & Srikanth, 2021).

In the article that follows, I review the literature on these questions herein, and address implications for policy and future empirical research. Next, I conduct a series of empirical tests using the National Education Cost Model to identify the optimal cost model, with respect to race sensitive and race neutral alternative cost factors.

Race & The Estimation of Costs of Equal Educational Opportunity

School finance formulas are broadly designed to promote equal educational opportunity, but rarely sufficiently calibrated to accomplish that goal. That is, they include basic funding levels (foundation levels) and cost adjustments or weights related to student needs and other factors, but the basic funding level is rarely if ever set based on the costs for students to meet a specific outcome standard. Similarly, the various weights and cost factors are rarely calibrated according to how much more it might cost to achieve the same outcomes, for some children than others, in one setting versus another. School finance formulas include a relatively standard set of student need and cost factors:

- All 50 states provide some adjustment or supplemental aid for children with disabilities;
- 44 states provide some adjustment or supplemental aid for children from economically disadvantaged backgrounds, with 26 providing a single weight and 13 providing multiple weights;
- 48 states provide some adjustment or supplemental aid for children for whom English is a second language with 27 states including single weights and 10 including multiple weights (Atchison et al., 2023).

Additionally, states allocate specific resources or provide block grants targeted toward serving specific student populations. None include weights, resource allocations or targeted grants based on student race or school or district racial composition.

In the 1990s and early to mid-2000s, several states engaged in analyses of the costs of programs and services needed to achieve existing state standards, mostly using panels of experts and practitioners to prescribe the programs and services needed (Baker, 2006, 2005). None of these analyses suggested different resources (smaller classes or different competitive wages) in relation to student enrollment racial composition. More recently, several states have engaged in statistical modeling to identify costs to achieve desired outcome goals, and how those costs vary by setting, location and students served (Taylor et al., 2018; Duncombe & Yinger, 2006; Kolbe et al. 2019; Baker et al., 2020; Atchison et al., 2023). Among these, one study in Delaware included school racial composition (% Black) as a factor (Atchison et al., 2023).

Among recent peer reviewed applications using education cost modeling, only those by Baker and colleagues, using a recent National Education Cost Model (NECM) include the role of race, with some comparing model estimate differences between models including and those not including racial composition (Baker, Weber & Srikanth, 2021; Levin et al., 2022; Kolbe et al., 2021; Zhao, 2022; Gronberg, Jansen & Taylor, 2017). Similarly, among older peer reviewed education cost modeling literature, only a handful of papers by Baker and colleagues addressed the role of race in affecting the costs of providing equal educational opportunity (Duncombe & Yinger, 1997, 1998; Imazeki & Reschovsky, 2006; Gronberg, Jansen & Taylor, 2012, 2011; Baker, 2011).

This includes a 2011 article by Baker in which he tests the effectiveness of race-neutral alternatives, including an interaction between poverty and population density to determine whether those race neutral alternatives are sufficient substitutes for including race directly in the model. This particular piece draws on related work by Duncombe (2007) and Duncombe and Yinger (2006). As noted at the outset of this article, William Duncombe (2007), while working on expert testimony on behalf of intervenor district St. Louis City, in a case on Missouri school funding, found it inescapable to include the district's racial composition in his cost estimates. A year earlier, Duncombe and colleague John Yinger (2006) had been contracted to conduct similar analyses for the State of Kansas to inform reforms of that state's school funding formula, which was under judicial oversight at the time. In that study, Duncombe and Yinger found their way around using district racial composition by instead including an interaction between poverty and population density to capture the higher costs in the state's more densely populated larger urban centers and towns which also served larger student minority populations. As a result of that analysis, the Kansas legislature adopted a special weight on child poverty in higher population density districts (Baker, 2022).

The modern era of statistical modeling of education costs traces back to the early 1990s, when Downes and Pogue estimated cost models on Arizona school districts to determine the additional costs of serving children from economically disadvantaged backgrounds. But long before that, Garms and Smith, in 1970 sought to identify the additional costs of mitigating racial and economic disparities in children's outcomes that had been identified in the Coleman Report a few years earlier (Garms & Smith, 1970). While not directly estimating a cost model by modern specifications, Garms and Smith did infer from analyses of the relationship between population characteristics and student outcomes, the weights that should be assigned to school funding in order to close achievement gaps – that is, provide equal educational opportunity. Garms and Smith modeled data on elementary school students in New York state. Among other things, they found: “We immediately found that it was necessary to keep the variables representing ethnic status (N and PR) in spite of the fact that they do not completely meet the criteria for a variable.” (Garms & Smith, 1970, p. 313) That is, it didn’t seem conceptually appropriate that race should be a determinant of cost, but it was statistically unavoidably so.

State Specific Cost Modeling & the Role of Race

There exist a handful of studies that have tested the role of race in education cost models – most specifically testing whether Black enrollment shares in schools or districts affect the costs of achieving any specific aggregate outcome(s). These studies test the importance of including racial composition by applying a handful of analytic techniques, asking:

1. Is school or district racial composition a statistically significant predictor of spending associated with achieving a specific outcome?
2. How does including district racial composition affect cost predictions for districts of varying racial composition? That is, are the difference of policy relevant, important magnitude?
3. Does omitting racial composition create omitted variables bias observable in model residuals?
4. Does omitting racial composition compromise model predictive validity?
5. Are race neutral alternatives, such as Duncombe and Yinger’s “poverty x density” interaction term sufficient substitutes?

On the first point, Baker and Green (2009) and Baker (2011) run a series of cost models in which they compare model estimates and results applying a) models excluding racial composition, b) models including racial composition (% Black, % Hispanic, % Black and Hispanic) and c) models including a race neutral alternative (Poverty x Population Density). In the first of these studies, modeling data from Arizona, New Jersey and Missouri, Baker and Green find: "In short, it costs more to achieve desired educational outcomes in school districts where larger shares of the student population are Black." But, "such findings appear less consequential in states such as Arizona with no majority Black school districts, failing to alter predictive validity of models." (p. 323) And finally, that different models worked better in different states, including whether interacting Black populations with population density improved model performance. In a related paper, focusing specifically on Missouri, Baker (2011) also found some conflicting evidence as to whether a race neutral model, interacting poverty and population density, was sufficient or whether inclusion of race was necessary, using the same, limited set of tests of out-of-sample model predictive validity. (p. 58) In each case, cost predictions were significantly affected for majority Black districts when including Black enrollment shares, but improvements to model predictive validity were varied, as noted above.

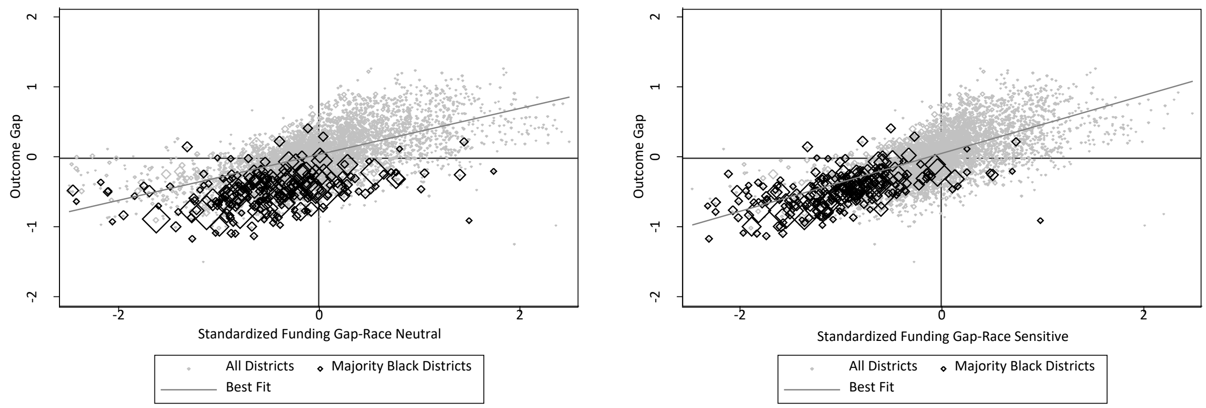
A more recent book chapter by Baker and Castillo (2024) used an updated version of the national cost model estimated by Baker, Weber and Srikanth (2021). Baker (2011) used data from 2001 to 2008 on Missouri school districts, with most data coming from Missouri Department of Elementary and Secondary Education sources. Baker and Castillo (2024) rely on a model using data on all districts nationally from 2009 to 2019, including federal data sources on spending (Census Fiscal Survey) and data on equated reading and math assessments from the Stanford Education Data Archive. Both show the influence of racial composition on cost predictions for a small overlapping set of majority Black Missouri school districts. Comparisons are provided in Table 1. Baker (2011) estimated cost index values (around a median cost of 1.0) for Missouri districts while Baker and Castillo (2024) report dollar values of predicted per pupil costs. Both find that when race is included in the model, cost predictions for these majority Black Missouri school districts go up by 20 to 37%. As such, failing to address racial difference would understate their costs by that amount.

Table 1.

	% Poverty	**% Black**	**Race Neutral Cost**	**Race Sensitive Cost**	**Ratio (Race Sensitive / Neutral)**
Center (58)	20%	61%	$13,352	$18,223	1.36
Hickman Mills	29%	72%	$16,404	$21,350	1.30
Kansas City	29%	57%	$18,968	$22,794	1.20
St. Louis	28%	79%	$16,698	$22,804	1.37
	Baker & Castillo (2024)				
	% Poverty	% Black	Race Neutral Cost Index	Race Sensitive Cost Index	Ratio (Race Sensitive / Neutral)
Center	19%	65%	1.00	1.23	1.23
Hickman Mills	19%	79%	1.00	1.32	1.32
Kansas City	28%	62%	1.01	1.21	1.21
St. Louis City	35%	81%	1.07	1.39	1.30
Baker (2011)					

Baker and Castillo (2024) explore the question of omitted variables bias in the national cost model, providing the following two figures (and supporting regression models). The figures show the relationship between "funding gaps" and outcomes, where "funding gaps" are measured as the difference between what districts presently spend versus what they are estimated to need to achieve average outcomes. As one might expect, those gaps are correlated with outcomes. The more adequate, per se, the funding, the higher the outcomes. Districts falling below the diagonal are districts that are underperforming with respect to their funding gap. That is, they are empirically *inefficient*. Either that, or their funding gap has been mismeasured? The left-hand panel shows that majority Black districts – identified as diamonds in the foreground – appear systematically inefficient. The right-hand panel shows that when Black enrollment shares are included in the cost model, majority Black districts fall along the diagonal and are no longer identified as inefficient. In cost analysis, inefficiency is something that can or should be controlled by district officials whereas cost factors are things outside of the control of district officials. Black enrollment shares and the history that created Black community and school racial isolation, leading to the observed effects in the figures below are cost factors outside the control of local district administrators and boards of education.

Figure 1



Baker and Castillo (2024) did not test whether race neutral alternatives could sufficiently resolve the omitted variables bias problem and Baker (2011) did not report a similar residual bias test but did show that predictive validity improvements (using split sample methods) were mixed.

Atchison and colleagues' (2023) school level model of costs to achieve state outcome standards in Delaware takes a different approach at testing and applying race neutral alternatives. Atchison and colleagues estimate a school level cost model using data provided by the state of Delaware. The cost predictions from this model across Delaware districts are correlated at .89 with adapted estimates from the National Education Cost Model, estimated with district level, national data. Atchison and colleagues employ a two-step process to get from their cost model to a weighted formula proposal. The cost model itself includes racial composition (% Black) as a factor. But in the second step, the authors test whether a parsimonious set of measures can be used to predict sufficiently the cost model predictions from the first step. They find that the race sensitive cost predictions can be accurately proxied with a race neutral model, predicting 98.1% of the variation in the cost estimates. Including race in the weights model increase the variance explained only marginally to 98.5% (See Table A1, Appendix A).

Finally, Di Carlo and Edmond (2024) provide a useful illustration of whether reducing school district fragmentation and racial isolation would alter cost variation across districts, using estimates from the National Education Cost Model for New Jersey school districts and simulating the effects of consolidating districts to the county level. They find: "If we hypothetically consolidated all the state's districts into their home counties, this would reduce racial/ethnic gaps in school funding adequacy by 75-80 percent;" and "The reduction is primarily due to the sharing of costs between districts serving vastly different student populations than their counterparts in the same counties."

This simulation cannot reveal whether aggregate costs would change, or be reduced by smoothing those costs out across districts within counties. Given the available data, the aggregate costs would stay the same because the cost model includes only a linear direct effect of racial composition on costs, as do other cost models discussed herein. Total costs are only reduced by defragmenting if costs escalate non-linearly with Black enrollments, or by interaction with other terms such as poverty. That said, balancing costs across jurisdictions itself is sufficient reason to pursue integration and district defragmentation.

Next Steps & Future Empirical Questions

The above studies and reports provide a patchwork of evidence regarding the importance of considering school or district racial composition in determining the costs associated with providing equal educational opportunity and in translating those costs into school funding formulas. The National Education Cost Model, and underlying data, along with the increased number of state specific analyses recently released and ongoing provide opportunities to fill in this patchwork and address some key remaining questions.

First, drawing on what has already been produced, both in terms of findings and methods/applications, further evidence on the extent to which school district racial composition affects costs of providing equal educational opportunity is warranted, including:

a) Whether different measures of racial composition, racial groups and subgroups matter and differ by state and regional policy context?
b) Whether non-linear relationships and interactions exist wherein reduction of racial isolation would lead to lower overall costs?
c) Whether and to what extent race-neutral alternatives are sufficient substitutes in state school finance policies?

Future empirical analyses addressing these questions should include *all of the above* methods for evaluating reliability and validity, including:

1) Predictive validity tests of models using split-cross validation techniques (training set, test set) as used by Baker (2011) wherein optimal model identification is based on maximizing prediction accuracy and minimizing prediction bias;
2) Residual bias checks for omitted variables bias as used by Baker and Castillo (2024);

3) Inter-model correlations between models estimated with alternative sources of data (federal vs state), alternative breadth of samples (single stage versus national or regional) and alternative units of analysis (district vs school);
4) Tests of race-neutral proxies using a second stage analysis for translation to policy:
    a. tests of "weights" models for explaining cost variation, and evaluation of residual bias;
    b. tests of predictive accuracy of "weights" models.

Perhaps the most effective test of whether consolidation and integration, or racially integrative defragmentation can affect overall costs is to estimate continuously updated cost models through a period of reform involving either fragmentation or defragmentation resulting in increased or decreased racial isolation. Repeated model estimation and updating may pick up changes to overall costs of meeting common outcome goals that cannot be picked up by way of modeling non-linearities and interaction terms with models for fixed time periods on systems experiencing minimal structural or demographic shift.

Empirical Tests of a National Education Cost Model

Here, I provide a limited series of empirical tests and illustrations using the National Education Cost Model, first published in 2021 (Baker, Weber & Srikanth, 2021). The model has since been updated with additional years of expenditure and outcome data. As described in our 2021 article, the model combines data from the School Finance Indicators Database, which includes district level financial data from the Census Fiscal Survey of Local Governments (F-33), demographic and economic context data from the EDGE data system of the National Center for Education Statistics, as well as the more recently developed Neighborhood Poverty Index, and our own extended version of the Education Comparable Wage Index. Our outcome measures are an overall outcome index, nationally normed and mean centered from the Stanford Education Data Archive, v5.0 (Reardon et al., 2024). These are the data sources that have only recently enabled estimation of a national cost model, and while imperfect (neither spending data nor outcome data, in my opinion, are perfectly, precisely equated across state boundaries), they provide a unique opportunity for multistate, interstate comparisons.

Similar to Baker and Green (2009) I test a set of models that include and exclude race variables:

1) race sensitive model including % Black enrollment;
2) race neutral model;
3) race neutral model with poverty rates interacted with population density;
4) race sensitive model interacting % Black with population density.

Earlier publications had not previously tested the fourth of these models. Of primary interest is whether the third model sufficiently captures the higher costs associated with racially isolated, often inner urban fringe or urban core districts, when compared with the race sensitive models.

Here, I test the above measures using a conventional 2-stage least squares (Instrumental Variables) education cost function, following our previous work and that of Duncombe and Yinger (Baker, 2006; Duncombe & Yinger, 2011; Kolbe et al., 2021). The model includes per pupil spending as the dependent variable, includes a handful of indirect controls for inefficiency, and treats the outcome measure as endogenous, using as (exogenous) instruments, the demographic characteristics of surrounding school districts (other than the observed).

I evaluate the four models above across four categories to identify the optimal model and then compare cost predictions from the models, specifically for districts whose enrollments are majority Black. These tests draw on the work of Baker (2006) and Duncombe (2006) where both laid out frameworks for evaluating the reliability, validity and usefulness of education cost analyses.

Category 1: Model Estimates & Diagnostics

First, the models must meet the basic statistical tests for model coefficients and tests that the instruments are valid (Partial F >10) and pass tests for overidentification (p-value of Hansen J>.05).

Category 2: Out of Sample Prediction

For this test, I split our modeling data set into an 80% sample and 20% sample, using a random seed. We fit models to the 80% sample (parameters reported in Appendix A, Table A2) and then use those models to predict the spending per pupil of the 20% extract. We compare both the absolute percent error (average forecast accuracy error) and the average percent error (which picks up over or under prediction bias) for the four models. Error rates in a national model of school district spending are expected to be quite high because I have tried to identify a sufficiently generalizable cost model (cost factors related to spending) and have not removed state specific effects. The question at hand is which model best predicts (lowest error) spending variation in an out of sample test.

Category 3: Model Validity – Input Gap to Outcome Gap

A basic validity check on cost estimates is to compare the funding gap – current spending to cost estimate for a given outcome – to the actual outcome gaps with respect to that same outcome. That is, do districts that spend more than needed to achieve a given outcome, also achieve more, and vice versa. More importantly, how strong is this correlation between funding gaps and outcome gaps. We compare our models on this basis. (Table A3, Appendix A)

Category 4: Omitted Variables Bias

From the previous analysis one can also check whether certain subsets of districts or whether districts on certain dimensions fall out of line from the average trend for the relationship between funding gaps and outcome gaps. For example, do I find that districts with majority Black enrollments on average have larger outcome gaps than expected, given the estimated size of their funding gaps? If so, our models may negatively bias the spending predictions for these districts. For this analysis, I estimate a model of the residuals from the relationship in Category 3 – the residuals of the relationship between funding gaps and outcome gaps – to determine whether those residuals are biased by district racial composition, and which among our four models yields the least residual bias of this type?

Findings

Table 2 shows the estimates and diagnostics for the four models. The vast majority of estimates are statistically significant and in the expected direction. The poverty by density, percent Black enrollment and Black by density coefficients are all significant. Models where percent Black is in the second stage, use percent Hispanic of surrounding districts as an instrument while models that exclude percent Black in the second stage include percent Black or Hispanic of surrounding districts as an instrument. Partial F statistic indicating instrument validity (that they predict variation in the outcome in the first stage model) are all sufficiently high (over 150). Overidentification may be an issue in the third model – poverty by density – which has a Hansen J p-value of <.05. Appendix A1 shows the estimates from the same models estimated to 80% of the data, revealing consistent findings. Results are similar when selecting a randomized 50% sample. In general, the models work, statistically speaking and are quite robust.

Table 2. Alternative Models

	IV Reg Grade 3-8 Black		**IV Reg Grade 3-8 Neutral**		**IV Reg Grade 3-8 Pov x Dens**		**IV Reg Grade 3-8 Black x Dens**	
	coef	**R.S.E**	**coef**	**R.S.E**	**coef**	**R.S.E**	**coef**	**R.S.E**
Outcome Index	1.841***	0.114	1.956***	0.127	1.935***	0.129	1.783***	0.106
Labor Cost Index	0.478***	0.035	0.573***	0.036	0.633***	0.037	0.499***	0.034
Student Needs								
Child Poverty Rate	2.260***	0.164	3.312***	0.227	2.312***	0.215	2.219***	0.155
Disability Rate (state ctr)	3.002***	0.148	2.957***	0.157	2.932***	0.158	2.929***	0.141
% ELL	1.938***	0.136	1.700***	0.133	1.622***	0.133	1.846***	0.126
% Black	1.034***	0.061					0.399***	0.069
%Poverty x Pop Density					0.217***	0.038		
%Black x Pop Density							0.115***	0.012
Grade Range Distribution								
% Pre-k	0.011	0.144	0.173	0.153	0.129	0.152	-0.007	0.140
% 9 to 12	0.507***	0.041	0.499***	0.044	0.498***	0.044	0.491***	0.040
Economies of Scale								
Less than 100 Students	0.573***	0.080	0.545***	0.082	0.551***	0.081	0.557***	0.079
101 to 300 Students	0.385***	0.022	0.367***	0.022	0.366***	0.021	0.371***	0.021
301 to 600 Students	0.220***	0.016	0.199***	0.016	0.199***	0.016	0.213***	0.016
601 to 1200 Students	0.141***	0.013	0.119***	0.013	0.117***	0.013	0.140***	0.012
1201 to 1500 Students	0.109***	0.014	0.090***	0.015	0.091***	0.015	0.112***	0.014
1501 to 2000 Students	0.100***	0.013	0.083***	0.013	0.085***	0.013	0.104***	0.012
Population Density (ln)	-0.061***	0.007	-0.036***	0.006	-0.076***	0.010	-0.070***	0.007
Efficiency Measures								
% Pop 5 to 17 Years Old	-1.004***	0.166	-0.444***	0.122	-0.429***	0.122	-0.975***	0.160
Housing Value Ratio	-0.356***	0.030	-0.409***	0.034	-0.380***	0.033	-0.329***	0.028
Herfindahl Index	-0.359	0.475	0.363	0.506	-0.005	0.499	-0.161	0.462
Time Period	0.016***	0.001	0.016***	0.001	0.014***	0.001	0.015***	0.001
Constant	8.560***	0.065	8.233***	0.063	8.314***	0.064	8.558***	0.063
Number of observations	147,186		147,186		147,186		147,186	
Instrument Diagnostics								
Partial F (excluded inst)	177.54		160.12		157.19		191.96	
Hansen J (p-value)	0.6245		0.3392		0.0406		0.7952	
Model Selection Tests								
Test Set (out of sample) Prediction								
Correlation (r) with current spending (20% Test Set)	.298		.271		.291		**.339**	
MAPE (20% Test Set)	36.2%		38.3%		37.2%		**35.1%**	
MPE (20% Test Set)	**14.0%**		17.6%		18.9%		15.2%	
FundGap x OutGap Validity Test (rsq)	0.450		0.383		0.391		**0.453**	
Residual Bias (% Black)	**0.055*** **		-0.417***		-0.394***		0.092***	

note: *** p<0.01, ** p<0.05, * p<0.1

Across the models, the rank order of performance on model selection tests is as follows:

1. For correlation between predicted out of sample spending and actual, Black by Density performed best, Black alone second, Poverty by Density third, and Poverty alone last;
2. For overall error rates of prediction of out of sample spending, Black by Density performed best, Black alone second, Poverty by Density third, and Poverty alone last, but for directional bias in out of sample prediction Black alone slightly outperformed Black by Density;
3. For the strength of the relationship between estimated funding gaps and outcome gaps, Black by Density performed best, Black alone second, Poverty by Density third, and Poverty alone last, but for racially correlated residuals (bias) Black alone performed slightly better than Black by Density.

That is, across all tests it is clear that racial composition is important to the prediction of costs and estimation of funding gaps, in terms of identifying the best – statistically speaking – model. It also seems relatively clear that interacting race with population density yields marginal improvements to the models.

Table 3 shows the differences in per pupil cost predictions (to achieve national average outcomes in reading and math, grades 3 to 8) for the Nation's majority Black enrollment districts which have over 2,000 enrolled pupils. That is, what is the expected per pupil cost for all children to have equal educational opportunity to achieve the relatively low bar outcome of national mean assessment scores? Many of these districts, like Birmingham City are specifically noted in Baker, Di Carlo and Green's 2022 report which outlined over a century of residential discrimination, district boundary manipulation and other factors that have made these districts the racially isolated districts they are today. Birmingham City is reported to be 88.7% Black. If I estimate the cost of providing students in Birmingham City to achieve the outcome goal in question, without including racial composition, I estimate that they would need to spend about $27,328 per pupil. If I try the race neutral alternative of interacting their high poverty rate with population density, that figure goes up slightly to $27,857. But, if I account for the share of the student population that is Black the cost estimate rises to nearly $44,000 per pupil, and interacted with density, just over $44,000 per pupil. The difference between the race neutral and race sensitive models is the *reparatory margin* needed to begin counteracting over a century's worth of racially discriminatory policies (Baker, Di Carlo & Green, 2022). *Reparatory margins* are smaller for the other cities in Table 3, but still of important magnitude.

Table 3. Differences in Cost Predictions – Majority Black School Districts (>20k enrollment)

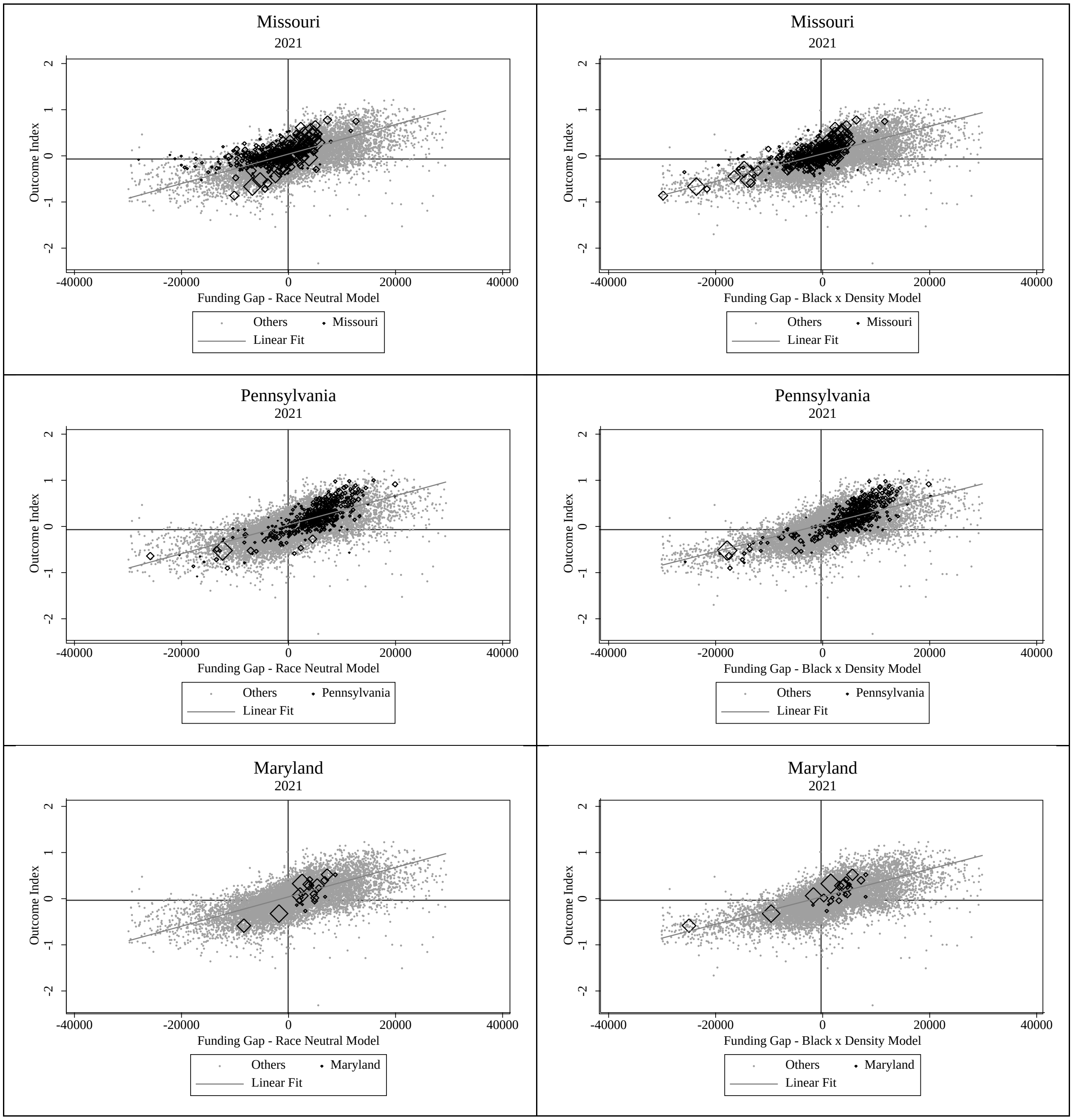
State	District	Enrollment	% Poverty	% Black	% Black	Neutral	Pov x Density	Black x Density
AL	**Birmingham City**	21,597	41.1%	88.7%	$43,960	$27,328	$27,857	$44,216
AL	Jefferson County	35,336	22.2%	50.8%	$21,235	$15,832	$16,104	$21,441
AL	Mobile County	52,460	23.1%	50.6%	$20,274	$14,998	$15,211	$20,189
AL	Montgomery County	27,399	32.5%	78.5%	$31,264	$19,425	$19,540	$30,683
AR	Little Rock SD	21,612	26.7%	60.6%	$30,482	$20,600	$20,756	$30,269
DC	District of Columbia PS	49,896	25.0%	57.7%	$28,354	$22,949	$24,062	$31,042
GA	Atlanta Public Schools	51,012	26.6%	72.2%	$25,524	$16,939	$17,491	$26,948
GA	Bibb County	21,373	36.3%	77.7%	$31,182	$20,842	$21,270	$31,624
GA	Savannah-Chatham County	36,502	23.9%	57.7%	$21,700	$16,062	$16,422	$22,051
GA	Clayton County	52,149	26.9%	69.2%	$28,101	$18,941	$19,431	$29,500
GA	DeKalb County	93,470	23.7%	59.3%	$26,836	$19,015	$19,422	$28,140
GA	Douglas County	25,884	19.5%	54.9%	$21,252	$15,466	$15,796	$21,611
GA	Henry County	42,388	12.2%	57.6%	$16,648	$11,072	$11,323	$17,040
GA	Muscogee County	30,757	30.4%	57.2%	$25,767	$19,013	$19,427	$26,212
GA	Richmond County	29,093	31.5%	74.9%	$28,468	$18,508	$18,897	$28,872
LA	Caddo Parish	36,153	29.8%	63.2%	$23,498	$16,926	$17,128	$23,317
LA	East Baton Rouge Parish	40,283	27.7%	71.3%	$26,286	$16,772	$17,055	$26,998
MD	**Baltimore City PS**	77,856	31.3%	75.7%	$38,322	$25,775	$27,089	$42,404
MD	Charles County PS	26,768	8.9%	56.7%	$15,764	$11,249	$11,560	$15,945
MD	Prince George's County	131,646	14.8%	55.3%	$25,800	$18,929	$19,305	$26,790
MI	Detroit Public Schools	48,782	40.2%	81.6%	$47,270	$31,302	$32,454	$50,440
MS	Jackson Public School District	20,401	40.1%	95.0%	$37,194	$21,282	$21,408	$36,389
NY	Rochester City SD	24,898	38.9%	53.4%	$47,071	$39,628	$40,612	$47,326
OH	Cincinnati Public Schools	34,635	32.2%	61.7%	$34,619	$26,288	$27,229	$36,016
OH	Cleveland Municipal	34,941	39.1%	64.0%	$47,169	$36,442	$37,953	$49,259
OH	Columbus City School District	46,657	29.2%	53.0%	$34,343	$26,577	$27,219	$35,337
PA	Pittsburgh SD	21,407	24.4%	51.3%	$22,171	$16,984	$17,472	$22,979
SC	Richland 01	22,202	26.6%	70.3%	$24,153	$15,509	$15,758	$24,348
SC	Richland 02	27,761	15.3%	61.0%	$17,568	$11,034	$11,197	$17,778
VA	Newport News PS	27,113	22.7%	54.1%	$20,388	$15,179	$15,643	$21,498
VA	Norfolk City PS	27,955	21.4%	58.4%	$20,397	$14,447	$14,973	$21,941
VA	Richmond City PS	28,225	34.1%	55.4%	$28,886	$23,756	$24,630	$30,449
WI	Milwaukee School District	71,510	30.4%	50.4%	$32,899	$27,055	$28,082	$34,411

Figure 2 provides illustrations for three states that are home to substantial racial segregation due to discriminatory housing polices. Model results in the left-hand panel are from the Poverty by Density, race neutral models and model results in the right-hand panels are from the Black by Density model. National results are in the background (gray dots) and the overall trendline of funding gaps to outcome gaps is included. Districts from the specific state are represented as Black Diamonds in the foreground. The size of the diamonds represents district enrollment. Districts falling below the diagonal are districts that achieve less than expected on the outcome index, given their relative funding. One might label them "inefficient." Or, as noted previously, one might consider the possibility that the models on the left suffer from omitted variables bias – that I've missed an important cost factor. Identifying and including that cost factor might move those districts *into line*, per se, with average efficiency expectations.

This is exactly what I see in Figure 2. Those larger diamonds in Missouri which fall well below the expected outcomes, and have race neutral funding gaps under $10,000 per pupil have much larger funding gaps when considering race, near and exceeding $20,000 per pupil and in the right-hand panel fall right on the average efficiency trajectory. That is, they move to the left (larger funding gap estimate) in the figure. The City of Philadelphia (largest district in Pennsylvania), in the second row, shows a similar though smaller movement from below the line, to right in line with average efficiency. The case is similar for the City of Baltimore (Diamond furthest to the left in the bottom row).

Figure 1.

State Specific Illustrations



Conclusions & Policy Implications

Prior studies and the analyses presented herein make a compelling case that school district racial composition is an important factor determining the costs of providing equal educational opportunity. First, models including racial composition reduce or eliminate residual bias, indicating that race-neutral models suffer from omitted variables bias. When that bias is reduced, districts with majority Black enrollments fall in line with average efficiency (in the production of outcomes) expectations. We should have no reason, other than perhaps our own racial biases, for hypothesizing differently. Perhaps most importantly, the predicted costs of providing equal educational opportunity, when including racial composition, change substantively for majority Black districts. Those districts, many of which have been created to be and reinforced as economically depressed and racially isolated for over a century, face uniquely higher costs of providing equal educational opportunity.

It's important to understand that, knowing the effects found herein – the magnitudes of the *reparatory margins* estimated herein – if we continue to ignore those effects and conduct only race avoidant analyses to guide race avoidant policy, we are knowingly depriving students of equal educational opportunity on the basis of their race. Ignoring the above findings deprives Black students in majority Black districts of equal opportunities to achieve common outcome goals – to succeed on state standards – perpetuating racial educational inequality.

A next step in this process is to determine whether and to what extent these costs and the total costs to a state of providing equal educational opportunity can be reduced substantively by defragmentation of racially isolated local public school districts to improve racial integration. Much has been made in the literature over the potential cost savings associated with consolidation of local public school districts, with emphasis on achieving economies of scale, especially for highly fragmented districts in close proximity to one another. The racial isolation that persists across many of these same boundaries and its affects on the costs of providing all children equal educational opportunity deserves at least equal attention and can be studied by similar methods. We expect, based on our finding herein that the costs associated with the interaction between racial composition and population density can be disrupted and reduced by district defragmentation and racial integration. If so, the efficient path toward providing racial equal educational opportunity is a both-and path, involving both appropriately calibrated state school finance systems to address all costs and needs and aggressively integrating and defragmenting local public school districts, to more efficiently achieve these ends. Finally, I see these efforts as both required under state constitutions that demand the provision of equal educational opportunity and as reparations for the decades of policies and practices that have inflicted these damages, and high costs on our nation's majority Black communities in particular.

Appendix A

Table A1

Weights models estimated to Delaware Data

Student Need Factor	Cost Model Coeff.		Weights 1 (Race Sensitive)		Weights 1 (Race Neutral)		Weights 2 (Race Sensitive)		Weights 2 (Race Neutral)	
% Low Income	0.520	*	1.696	*	1.966	*	1.696	*	1.966	*
% with Disabilities	1.159	*	3.157	*	3.075	*	3.157	*	3.075	*
% Complex/Multiple Disabilities	1.396	*	4.101	*	4.288		4.101	*	4.288	
% ELL	0.260	*	1.280	*	1.140	*	1.280	*	1.140	*
% Voc Tech	1.535	*	4.620	*	4.594		4.620	*	4.594	
% Middle Grades	-0.011		0.987	*	0.978	*	0.987	*	0.978	*
% Secondary grades	0.038		1.036	*	1.033	*	1.036	*	1.033	*
% Black	0.149	*	1.159	*			1.159	*		
R-Squared			0.9831		0.9791		0.9851		0.9811	

*p<.05

Table A2

Training Set Models (80% Sample)

	IV Reg Grade 3-8 Black		IV Reg Grade 3-8 Neutral		IV Reg Grade 3-8 Pov x Dens		IV Reg Grade 3-8 Black x Dens	
	coef	**R.S.E**	**coef**	**R.S.E**	**coef**	**R.S.E**	**coef**	**R.S.E**
Outcome Index	1.866***	0.118	1.987***	0.132	1.965***	0.135	1.808***	0.110
Education Comparable Wage Index	0.478***	0.035	0.575***	0.037	0.636***	0.037	0.499***	0.034
Student Needs								
Child Poverty Rate	2.304***	0.170	3.377***	0.237	2.340***	0.221	2.261***	0.161
Disability Rate (state ctr)	3.010***	0.152	2.977***	0.163	2.949***	0.163	2.936***	0.145
% ELL	1.959***	0.140	1.724***	0.137	1.643***	0.137	1.868***	0.130
% Black	1.046***	0.063					0.411***	0.071
%Poverty x Pop Density					0.224***	0.040		
%Black x Pop Density							0.115***	0.012
Grade Range Distribution								
% Enrollment in Pre-k	-0.012	0.144	0.155	0.155	0.110	0.154	-0.030	0.140
% Enrollment in Secondary Grades	0.500***	0.042	0.492***	0.045	0.490***	0.045	0.484***	0.041
Economies of Scale								
Less than 100 Students	0.547***	0.080	0.518***	0.081	0.525***	0.080	0.532***	0.078
101 to 300 Students	0.382***	0.022	0.365***	0.022	0.364***	0.022	0.368***	0.021
301 to 600 Students	0.220***	0.016	0.200***	0.016	0.200***	0.016	0.213***	0.016
601 to 1200 Students	0.142***	0.013	0.120***	0.013	0.119***	0.013	0.141***	0.012
1201 to 1500 Students	0.112***	0.014	0.094***	0.015	0.095***	0.015	0.115***	0.014
1501 to 2000 Students	0.098***	0.013	0.081***	0.014	0.084***	0.014	0.103***	0.013
Log of Population per Square Mile	-0.063***	0.007	-0.037***	0.006	-0.078***	0.011	-0.072***	0.007
Efficiency Measures								
% Pop 5 to 17 Years Old	-1.019***	0.160	-0.442***	0.122	-0.428***	0.122	-0.990***	0.154
Housing Value Ratio	-0.365***	0.031	-0.420***	0.036	-0.390***	0.035	-0.338***	0.029
Herfindahl Index	-0.353	0.480	0.355	0.518	-0.024	0.510	-0.151	0.468
Time Period	0.016***	0.001	0.016***	0.001	0.014***	0.001	0.015***	0.001
Constant	8.570***	0.066	8.237***	0.065	8.322***	0.066	8.568***	0.064
Number of observations	117,656		117,656		117,656		117,656	

note: *** p<0.01, ** p<0.05, * p<0.1

Table A3

Residual Bias Check (with year fixed effect)

DV=Model Residuals	**% Black**		**Neutral**		**Pov x Density**		**Black x Density**	
	coef	**se**	**coef**	**se**	**coef**	**se**	**coef**	**se**
% Black	**0.055*** **	0.004	-0.417***	0.004	-0.394***	0.004	0.092***	0.004
% Hispanic	-0.087***	0.003	-0.037***	0.003	-0.029***	0.003	-0.083***	0.003
Census Poverty Rate 5 to 17 yr Olds	-0.820***	0.008	-0.635***	0.008	-0.627***	0.009	-0.851***	0.008
Constant	0.153***	0.003	0.188***	0.003	0.182***	0.003	0.151***	0.002
Number of observations	149,224		149,224		149,145		149,145	
R2	**0.111**		0.191		0.179		0.113	

note: *** p<0.01, ** p<0.05, * p<0.1

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